The 30th Chinese Physics Olympiad

Final — Theoretical Examination (Problems)

2013 — duration: 3 hours — 8 problems, 140 points

Notes to candidates (abridged). The examination lasts 3 hours and contains eight problems with a total score of 140 points. The solution of a calculation problem must show the necessary text explanations, equations, and the important steps of the work; a bare final result receives no credit. Where a numerical value is required, the answer must state both the number and the unit.

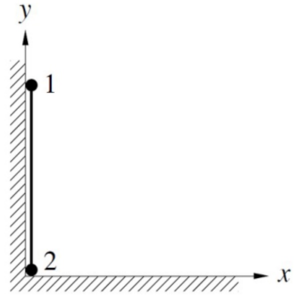
Problem 1 (15 points)

A small ball of mass m is thrown horizontally, with horizontal speed $\sqrt{2gh}$, from a point at height h above level ground. Air resistance is negligible. At every bounce from the ground the horizontal component of the velocity of the ball does not change, and the magnitude of the vertical component decreases each time by the same ratio. Counted from the first bounce, the total area enclosed between the trajectory of the ball and the projection of the trajectory on the ground is $\frac{8}{21}h^2$. Find the total impulse that the ground gives to the ball in all of the collisions.

Hint: for each flight of the ball that starts at the ground and ends at the ground (projectile motion), the area enclosed between the trajectory and its projection on the ground equals $\frac{2}{3}$ of the product of the maximum height and the horizontal range of that flight.

Problem 2 (15 points)

Two small balls 1 and 2, each of mass m, are connected by a rigid light rod of length l. The rod stands vertically in the corner between a smooth vertical wall and a smooth horizontal floor; ball 1 is at the upper end of the rod, as shown in the figure. Ball 2 is given a very small initial velocity to the right. During the motion of the system, at what angle between the rod and the vertical wall does ball 1 leave the wall?



Problem 3 (25 points)

A spacecraft in space uses its own propulsion to stay directly above a point of the Earth's equator, at height $L = \alpha R_e$, so that it remains at rest relative to a ground supply station on the equator.

Here R_e is the radius of the Earth, α is a constant with $\alpha > \alpha_m$, where

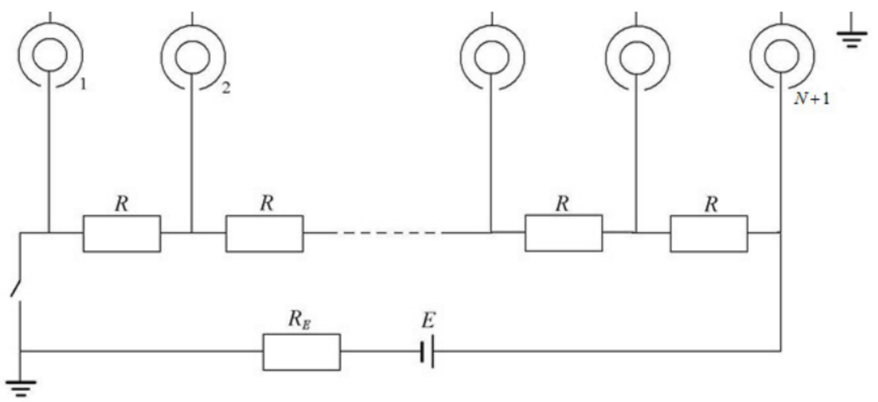
$$\alpha_m = \left(\frac{GM_e}{\omega_e^2 R_e^3}\right)^{1/3} - 1,$$

M_e and ω_e are the mass and the spin angular velocity of the Earth, and G is the gravitational constant. Suppose that a rigid transport tube of uniform wall and total mass m_p connects the supply station to the spacecraft; the tube is vertical, and its lower end is fixed to the ground. Assume that the tube always stays vertical. To send up goods, the goods are placed on a flat pallet inside the lower end of the tube and are pushed up along the tube; the speed of the pallet is kept small. Friction between the pallet and the tube wall is negligible. Take the spin of the Earth into account, but not the orbital motion of the Earth around the Sun. The total mass of the goods and the pallet in one delivery is m.

1. In the delivery of the goods from the station to the spacecraft, how much work is done by the gravitational force of the Earth, and how much by the inertial centrifugal force?
2. In the delivery of the goods from the station to the spacecraft, what is the minimum positive work that the external pushing force must do?
3. For what height of the spacecraft above the ground (call it L_0) is the sum of the work done by the Earth's gravity and the work done by the inertial centrifugal force, during the delivery, equal to zero?
4. Suppose the propulsion of the spacecraft is controlled so that, when no goods are being transported, the force between the spacecraft and the tube is always zero. Calculate the force that the ground supply station exerts on the tube when the height of the spacecraft is $L = \alpha R_e$ ($\alpha > \alpha_m$). Then, for the three cases $L > L_0$, $L = L_0$ and $\alpha_m R_e < L < L_0$, give the magnitude and the direction of the force that the supply station exerts on the tube.

Problem 4 (20 points)

A circuit contains a dc source of emf E and internal resistance R_E, and N identical resistors, each of resistance R. There are $N+1$ identical conducting spheres of radius r, connected into the circuit by long thin wires. To remove the mutual influence of the conducting spheres, each sphere is enclosed in a concentric grounded thin conducting spherical shell of inner radius r_0 ($> r$); each shell has a small opening that lets the thin wire enter without touching the shell, as shown in the figure. The spheres are numbered 1 to $N+1$ from the left. All spheres are initially uncharged. After the switch is closed and the steady state is reached, the total charge on the conducting spheres is Q. What is the radius of the conducting spheres? The electrostatic constant k is given.



Problem 5 (20 points)

As shown in the figure, a thin superconducting circular ring of radius r_0, mass m and self-inductance L is above a vertical cylindrical magnet, with its symmetry axis on the symmetry axis of the magnet. Near the ring the magnetic field has cylindrical symmetry, and its magnitude can be described approximately by a vertical component

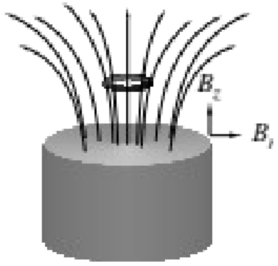
$$B_z = B_0 \, (1 - 2\alpha z)$$

and a radial component

$$B_r = B_0 \, \alpha r,$$

where B_0 and α are constants greater than zero, and z and r are the vertical and the radial position coordinates. At time $t = 0$ the center of the ring is at $z = 0$, $r = 0$, and the current in the ring is I_0 (the positive direction of the current in the ring and the positive z direction satisfy the right-hand rule). The ring is then released; it starts to move downwards, and its symmetry axis stays vertical. A thin superconducting ring has this property: the magnetic flux through the ring does not change.

1. What kind of motion does the ring perform? Give the dependence of the z coordinate of the ring center on time.
2. Find the expression for the current I in the ring at time t.



Problem 6 (15 points)

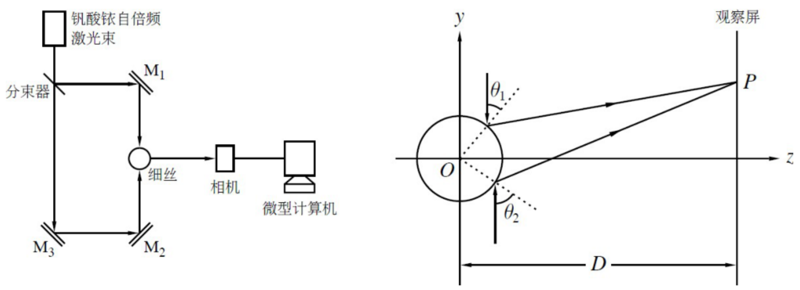
A thin metal disc of thickness t hangs in air of temperature 300.0 K. Its upper surface receives direct sunlight, and the temperature of the upper surface is 360.0 K; the temperature of the lower surface is 340.0 K. The temperature of the air does not change. The energy that each surface of the disc loses to the air in unit time is proportional to the temperature difference between that surface and the air, and to the area of that surface. Energy loss through the side (rim) of the disc is negligible. If the thickness of the disc is changed to 2 times the original value, find the temperatures of the upper and the lower surfaces of the disc.

Problem 7 (15 points)

The figure (a) shows a two-beam interference set-up that measures the diameter of a thin wire (sub-millimeter diameter). M_1, M_2, M_3 are fully reflecting mirrors, and the camera is a CCD (charge-coupled device) camera. The laser beam from a frequency-doubled Nd:YVO_4 laser is divided into two beams by the beam splitter: one beam is reflected by the mirror M_1 and then falls on the wire from the upper side; the other beam is reflected by the mirrors M_3 and M_2 one after the other and then falls on the wire from the lower side. Figure (b) shows the light paths of the two reflected rays that produce the interference: θ_1 and θ_2 are the angles of incidence on the upper and the lower side, D is the distance from the axis of the wire to the observation screen (the light-sensitive surface of the camera), and P is the point where the two reflected rays cross on the screen. A rectangular coordinate system is set up as follows: the origin O is on the axis of the wire; the x axis (not drawn) is along the axis of the wire and points into the paper; the y axis is parallel to the rays incident on the wire (positive direction up); the z axis points toward the observation screen and is perpendicular to it. The wavelength of the light is λ, and the spacing of the interference fringes on the screen is a. Because D is much larger than the wire diameter and the screen size, you can assume that only thin pencils of rays that travel nearly parallel to the z axis reach the screen.

1. Because the distance from the wire to the observation screen is much larger than the size of the screen, only the narrow pencils of the incident light with angle of incidence near 45°, on the upper and the lower side, are reflected by the wire onto the screen. The reflected pencils from the upper and the lower side form two virtual images of the source. Find the positions of these two virtual images. (*Note:* for $x \sim 0$, $\sin x \sim x$, $\cos x \sim 1$.)

2. Find the diameter d of the wire.



图(a) 图(b)

Labels in the figure: 钒酸钕自倍频激光束 = *beam of the frequency-doubled Nd:YVO$_4$ laser;* 分束器 = *beam splitter;* 细丝 = *wire;* 相机 = *camera;* 微型计算机 = *microcomputer;* 观察屏 = *observation screen;* 图 *(a)/(b) = figure (a)/(b).*

Problem 8 (15 points)

Relative to student Li, who stands on the ground, student Zhang moves to the right with relativistic speed v, and student Wang moves to the left with the same speed v. At the moment when Zhang and Wang meet, all three students set the readings of their own clocks to zero. At the moment when the distance between Zhang and Wang is L (observed in the ground reference frame), Zhang claps his hands once. The relative speed of Zhang with respect to Wang is

$$v_r = \frac{2\beta c}{1+\beta^2},$$

where $\beta = \dfrac{v}{c}$ and c is the speed of light in vacuum.

1. Find the reading of the clock carried by Zhang at the moment when Zhang claps.

2. In the rest frame of Wang, at the instant when the event “Zhang claps” happens, Wang also claps once. In the rest frame of Zhang, at the instant when the event “Wang claps”

happens, Zhang claps a second time. In the rest frame of Wang, at the instant when the event “Zhang claps the second time” happens, Wang claps a second time; and so on. Find the distance between Zhang and Wang, in the ground reference frame, at the moment when Zhang claps the n-th time.

3. In the rest frame of Li, what are the instants of the successive claps of Zhang and of Wang? State the order of these instants.