The 31st Chinese Physics Olympiad

Final — Theoretical Examination (Problems)

2014 — 8 problems

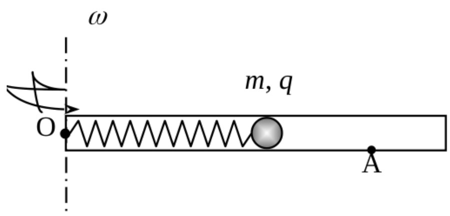
Problem 1

The figure shows the principle of a device that measures and controls the speed of rotation. A positive charge of magnitude Q is at the point O. A light, thin, insulating tube, smooth on the inside, can rotate in a horizontal plane about a vertical axis through the point O. Inside the tube, at distance L from O, there is a photoelectric trigger switch A. One end of a light insulating spring of natural length $L/4$ is fixed at the end O of the tube; its other end is connected to a small ball of mass m that carries a positive charge q.

Initially the system is in static equilibrium. Under an external torque the tube rotates about the fixed axis, and the ball can move inside the tube. When the rotation speed ω of the tube grows gradually, the ball arrives at the point A of the tube exactly in radial equilibrium relative to the tube, and it triggers the control switch: the external torque becomes zero at that instant (this prevents excessive rotation speed), and at the same time the charge at O changes into an equal negative charge $-Q$. The rotation speed can then be determined by a measurement of the new position B at which the ball is in radial equilibrium relative to the tube. The measured distance is $OB = L/2$. Find:

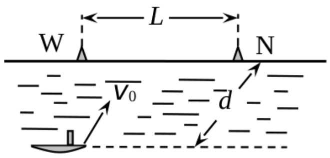
(1) the spring constant k_0, and the rotation speed of the tube when the ball is at B;

(2) whether the ball can perform small radial oscillations, relative to the tube, about the equilibrium point B. If such oscillations exist, find their period.



Problem 2

A multiple-warhead attack system is an effective means against a missile defence system. As shown in the figure, two military targets W and N are on a coastline, a distance L apart. A submarine patrols on a course parallel to the coastline and watches the two targets; the distance between its course and the coastline is d. After it receives the attack order, the submarine rises to the sea surface and launches a missile (the "mother bomb"), with launch speed v_0 (much larger than the patrol speed of the submarine). Assume that the missile separates into three warheads when it reaches the highest point of its flight. The three warheads have equal masses; at separation, the speed of each warhead relative to the original missile is v, and the three relative velocities lie in the same horizontal plane. Warheads 1 and 2 are real; warhead 3 is a decoy that confuses the radar of the opponent. If the two real warheads must hit the military targets W and N, find the position of the submarine at the moment of launch and the direction of the launch, and give the conditions under which the attack can be realized.



Problem 3

As shown in the figure, an adiabatic container is divided by two fixed adiabatic partitions into three chambers A, B, C of equal volume V_0 ($V_A = V_B = V_C = V_0$); the partitions carry the valves K_1 and K_2. The left end of the container is closed by an adiabatic piston H. Chamber A contains $\nu_1 = 1$ mole of a monatomic ideal gas in an equilibrium state with pressure P_0 and temperature T_0; the middle chamber B is a vacuum; chamber C contains $\nu_2 = 2$ moles of a diatomic ideal gas, whose measured equilibrium temperature is $T_c = 0.50\,T_0$. At the initial moment both K_1 and K_2 are closed. The system then goes through the following thermodynamic processes, in this order, with no heat exchange with the outside:

(1) Open K_1 and let the gas in V_A expand freely into the middle vacuum chamber V_B. After the gas reaches equilibrium, push the piston H slowly and compress the gas, so that the volume of chamber A becomes smaller by 30% ($V_A' = 0.70\,V_0$). Find the equilibrium temperature and pressure of this gas before and after the compression.

(2) Keep K_1 open and open K_2; let the two gases in the container mix freely and reach a common equilibrium state. Find the temperature and the pressure of the gas mixture at this point.

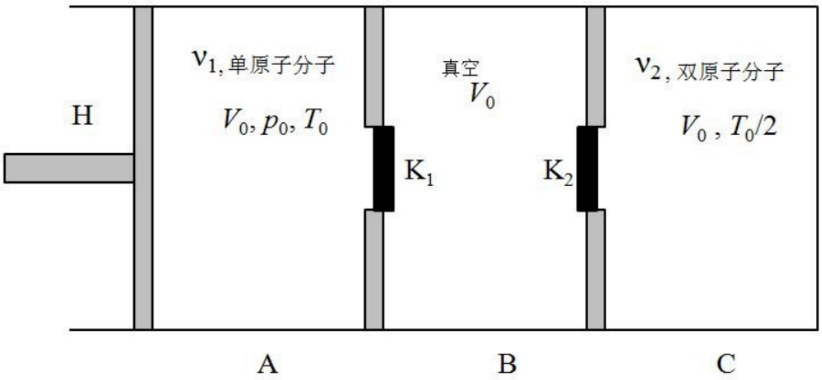
(3) With K_1 and K_2 both open, pull the piston H slowly, so that the volume of chamber A returns to its initial value $V_A'' = V_0$. Find the temperature and the pressure of the gas mixture at this point.

Hint: In all these processes the gases can be treated as ideal; numerical results may contain exponential or fractional expressions. By the second law of thermodynamics, when a thermodynamic system that consists of one ideal gas goes from an initial state (p_i, T_i, V_i) to a final state (p_f, T_f, V_f) through an adiabatic reversible process (a quasi-static adiabatic process), its state parameters satisfy the equation

$$(\Delta S_1)_{if} = \nu_1 C_{V_1} \ln\left(\frac{T_f}{T_i}\right) + \nu_1 R \ln\left(\frac{V_f}{V_i}\right) = 0, \tag{I}$$

where ν_1 is the number of moles of the gas, C_{V_1} is its molar heat capacity at constant volume, and R is the universal gas constant. When the thermodynamic system consists of two ideal gases, equation (I) must be replaced by

$$(\Delta S_1)_{if} + (\Delta S_2)_{if} = 0. \tag{II}$$



Labels: 单原子分子 = *monatomic molecules;* 真空 = *vacuum;* 双原子分子 = *diatomic molecules.*

Problem 4

A fiber grating is an optical device in which the refractive index of the medium changes periodically. Suppose the refractive index of the base material of the fiber core of a fiber grating is $n_1 = 1.51$; the refractive index of the core material is changed periodically along the fiber, and in the changed parts the refractive index is $n_2 = 1.55$. Layers of refractive index n_2 and thickness d_2 alternate with layers of refractive index n_1 and thickness d_1; the total number of layers is N. The longitudinal section is shown in figure (a). In the design of this device one considers, as a rule, only a single reflection at each interface, and the absorption of light in the medium is neglected. Assume the wavelength of the incident light in vacuum is $\lambda = 1.06\,\mu\text{m}$. When the reflected light interferes constructively, what minimum values must the thicknesses d_1 and d_2 of each layer have? If the reflectivity of the device must reach 8%, what is the minimum total number of layers N?

Hint: As shown in figure (b), when light passes with normal incidence from a medium of refractive index n_1 into a medium of refractive index n_2, reflection and transmission occur at the interface, and

$$\frac{\text{reflected field}}{\text{incident field}} = \frac{n_1 - n_2}{n_1 + n_2}, \qquad \frac{\text{transmitted field}}{\text{incident field}} = \frac{2n_1}{n_1 + n_2}, \qquad \text{reflectivity} = \left|\frac{\text{reflected field}}{\text{incident field}}\right|^2.$$

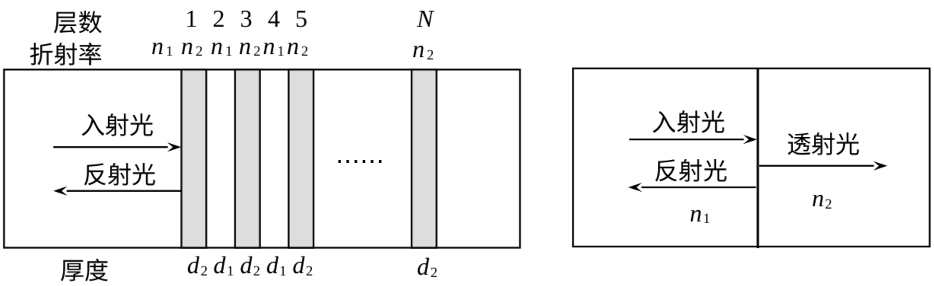


图 (a) 图 (b)

Labels: 层数 = *layer number;* 折射率 = *refractive index;* 入射光 = *incident light;* 反射光 = *reflected light;* 透射光 = *transmitted light;* 厚度 = *thickness;* 图 *(a)/(b) = figure (a)/(b).*

Problem 5

The Neutral-Particle Analyser (NPA) is an important instrument in nuclear-fusion research; it measures the temperature and the energy distribution of fast ions. Its principle is shown in the figure. Through measurements of the energy and the momentum of high-energy (200 eV – 30 keV) neutral atoms, which pass easily through the electromagnetic fields of the detection region, one can diagnose the properties of the ions that have collided sufficiently with these neutral atoms.

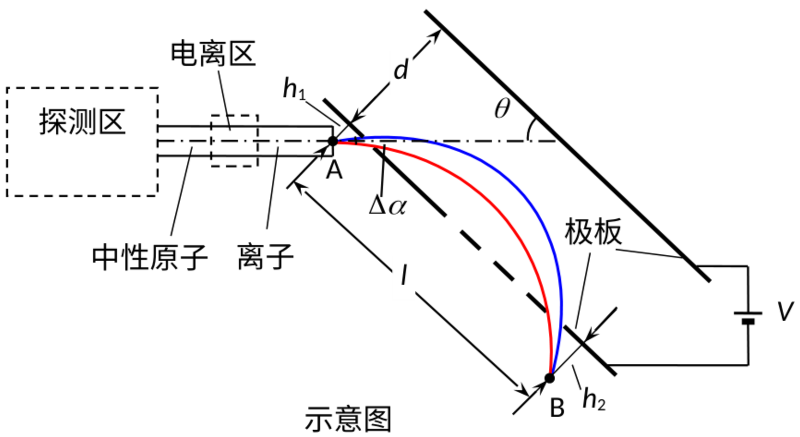
To measure the energy distribution of the neutral atoms, the neutral atoms are first ionized; the ion beam is then injected, at the angle θ, into the region between two parallel plate electrodes with plate separation d and voltage V. The ions are deflected by the electric field and then leave the field region. Under the condition that a measured ion does not touch the upper plate, the energy of the ion is determined through a measurement of the distance l, parallel to the plates, between the entry hole A and the exit hole B. The distance from A to the lower plate is h_1, and the distance from B to the lower plate is h_2; the charge of the ion is q.

(1) Derive the relation between the ion energy E and l, and give the maximum flight distance of the ion inside the plates in the direction perpendicular to the plates.

(2) A measured ion beam generally has an angular divergence $\Delta\alpha$ ($\Delta\alpha = \theta$). To improve the precision of the measurement, ions with the same energy E whose directions of incidence vary within the range $\Delta\alpha$ must leave through the same small hole B. Find the expression for h_2, and give the relation between the energy E and l in this case.

(3) To improve the energy resolution, an ion with the upper-limit energy of the measuring range must land exactly at the largest value $l_{\max}$ that the instrument allows; at the same time, to make the instrument small, the plate separation d must be as small as possible while the measurement requirements stay satisfied. Use the results of part (2) to find the expression for d. If $\theta = 30°$, what is the result?

(4) To distinguish the masses of these ions, design a follow-up device: give the principle diagram and the expression for the ion mass.



Labels: 探测区 = *detection region;* 电离区 = *ionization region;* 中性原子 = *neutral atoms;* 离子 = *ions;* 极板 = *plates;* 示意图 = *schematic diagram.*

Problem 6

An important application of superconductors is the winding of magnets that produce strong magnetic fields; the superconducting wires used for this belong to the type-II superconductors. When such a superconductor is placed in an external magnetic field of magnetic induction B_a, the magnetic field lines penetrate the superconductor in the form of flux quanta (also called flux vortex lines), and the flux quanta form a regular triangular flux lattice in the superconductor, as shown in figure 1. A flux quantum, shown in figure 2, has at its center a normal-state (nonzero-resistance) core region of radius ξ; around the core the material is in the superconducting state (zero resistance) and carries a superconducting current. The magnetic flux carried by one flux quantum is

$$\Phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15}\ \text{Wb}$$

(the smallest unit of magnetic flux).

(1) If $B_a = 5 \times 10^{-2}$ T, find the distance a between the flux vortex lines at this field.

(2) As B_a grows, the density of the flux vortex lines grows steadily; when B_a reaches a critical value B_{c2}, the whole superconductor becomes normal. Assume the radius of the vortex core is $\xi = 5 \times 10^{-9}$ m; find the corresponding critical field B_{c2}.

(3) In an ideal type-II superconductor, when a current I flows through a superconducting tape, the flux vortex lines move in a viscous flow under the driving of the Ampère force; this produces a resistance inside the tape (also called flux-flow resistance) and therefore Joule losses, which are harmful for the operation of a superconducting magnet. The speed v of the steady viscous flow of the vortex lines is related to the driving force f_A per unit volume of vortex line and to B_a by

$$f_A = \eta \frac{B_a}{\Phi_0} v,$$

where η is a proportionality constant. The directions of the applied field and of the current, and the dimensions of the superconducting tape, are shown in figure 3. State the direction of the flux-vortex flow, and find the flux-flow resistivity ρ (express it with b, c, d and the given quantities).

(4) How can one obtain a non-ideal type-II superconductor, in which the flux vortex lines do not flow easily? (Answer briefly.)

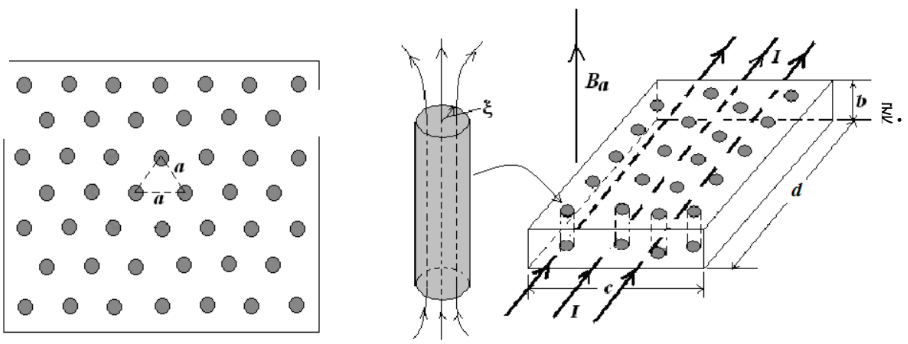


图 1 图2 图3

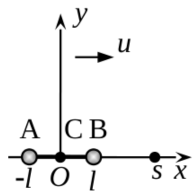
图 *1/2/3 = figure 1/2/3.*

Problem 7

As shown in the figure, two small balls A and B, each of mass m (both can be treated as point masses), are fixed at the two ends of a rigid light thin rod of length $2l$; the center of the rod is the point C. The system of the two balls and the rod is a particle system. Set up the coordinate system Oxy in a vertical plane: the direction Ox is horizontal, to the right; the direction Oy is vertical, upwards. Initially the center C of the particle system is at the origin O, and the system is thrown vertically upwards with initial speed v_0; during the ascent the line ACB stays horizontal. The wind speed is constant, equal to u, in the positive x direction. The air resistance f on a ball in motion is proportional, and opposite, to the velocity $\boldsymbol{v}$ of the ball relative to the moving air, that is $\boldsymbol{f} = -k\boldsymbol{v}$, where k is a positive constant. When the point C rises to its highest point, a small pebble (a point mass) of mass m_1 and speed u_1, moving in the positive y direction, collides with ball A in the vertical direction. The collision is perfectly elastic, and the collision time is extremely short. Afterwards the point C falls back to the same horizontal height at which the throw started; its position in the Ox direction at that moment is s. The time of the whole process, from the throw to the return, is T, and the number of full turns of the particle system in this process is n. Find, for the particle system:

(1) the initial angular velocity ω_0 of the rotation, and the relation between the angular velocity ω_s at the return to the point s and the number n;

(2) the work W_f done by the air resistance in the process from the start of the throw to the return to the point s, expressed with n, s and T.



Problem 8

The Sun is the star on which our life depends. Its main component is hydrogen; the hydrogen contracts under its own gravitation, and the temperature rises. When the temperature is high enough, the neutral atoms ionize into a plasma of protons and electrons; in the core region the temperature reaches about 1.5×10^7 K and the density reaches more than 1.6×10^5 kg/m^3, thermonuclear fusion sets in and releases an enormous amount of energy, which balances the gravitational contraction — a star is born. The main nuclear reactions inside the Sun are

$$^1\mathrm{H} + {}^1\mathrm{H} \ \rightarrow\ \mathrm{D} + \mathrm{e}^+ + \nu_e \tag{I}$$

$$\mathrm{D} + {}^1\mathrm{H} \ \rightarrow\ {}^3\mathrm{He} + \mathrm{x} \tag{II}$$

$$^3\mathrm{He} + {}^3\mathrm{He} \ \rightarrow\ {}^4\mathrm{He} + {}^1\mathrm{H} + {}^1\mathrm{H} \tag{III}$$

The probability of the first reaction is controlled by the weak interaction and is very small; exactly this lets the energy release out slowly. The positron e^+ produced in the reaction annihilates with an electron e^- into γ rays:

$$\mathrm{e}^+ + \mathrm{e}^- \ \rightarrow\ \gamma + \gamma. \tag{IV}$$

Given: the masses of the proton (^{1}H), the deuteron (D), helium-3 (^{3}He), helium-4 (^{4}He) and the electron are, respectively, 938.27, 1875.61, 2808.38, 3727.36 and 0.51 (MeV/c^2), with an uncertainty of 0.01 MeV/c^2; c is the speed of light in vacuum; the mass of the neutrino ν_e is smaller than 3 eV/c^2. The Planck constant is $h = 6.626 \times 10^{-34}$ J·s, $c = 3.0 \times 10^8$ m/s, the Boltzmann constant is $k = 1.381 \times 10^{-23}$ J/K, and the electron charge is $e = 1.602 \times 10^{-19}$ C.

(1) Use the ideal-gas model to estimate the mean kinetic energy of each kind of particle in thermal equilibrium, and the pressure in the core region of the Sun (give the two results in eV and in atm, respectively).

(2) Which particle (α, β, γ, p or n) is the particle x in reaction (II)? Calculate the kinetic energy and the magnitude of the momentum of this particle. Can one find a reference frame in which the kinetic energy of the particle x is zero?

(3) Give the ranges of the kinetic energies of the reaction products in reaction (I).

(4) The γ rays (photons) of reaction (IV) reach the surface of the Sun after very many Compton scatterings, with a wavelength of about 5.4×10^{-7} m at the surface. Suppose the energy of the incident photon is E_0, the energy of the photon after one scattering is E, and the scattering angle of the photon is ϕ (the angle between the momentum of the outgoing photon and the momentum of the incident photon). Derive the expression for E in terms of E_0 and ϕ. Assume that a photon goes through 10^{26} Compton scatterings on the way from the center of the Sun to the surface, and that the scattering angle is the same in every scattering; find the scattering angle ϕ of one scattering.