The 32nd Chinese Physics Olympiad

Final — Theoretical Examination (Problems)

2015 — 8 problems

This paper may need the following formulas:

$$\int_{x_1}^{x_2} \frac{\mathrm{d}x}{x} = \ln\frac{x_2}{x_1}; \qquad \int \frac{\mathrm{d}x}{\sqrt{x}} = 2\sqrt{x} + C; \qquad \ln(1+x) \approx x - \frac{x^2}{2} \ \text{ for } |x| \ll 1.$$

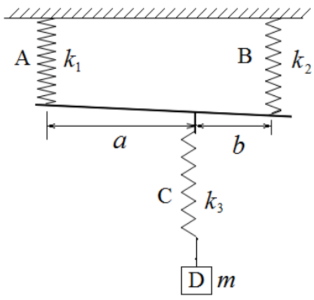
Problem 1

A light rod hangs from the ceiling on two light springs A and B, which are connected to its two ends; a block D is connected to the rod through a light spring C. The spring constants of A, B and C are k_1, k_2 and k_3, and the mass of D is m. The horizontal distances from the junction of C with the rod to the ends A and B are a and b. When the whole system is in equilibrium, the rod is nearly horizontal, as shown in the figure. Assume that the block D makes small oscillations in the vertical direction only, that A and B always stay vertical, and that air resistance is negligible.

(1) Find the extensions of the three springs, relative to their natural lengths, when the system is in its equilibrium position.

(2) Find the natural frequency of the small vertical oscillation of the block D.

(3) For what condition on a and b is the natural frequency of the block D largest? Find this largest natural frequency.



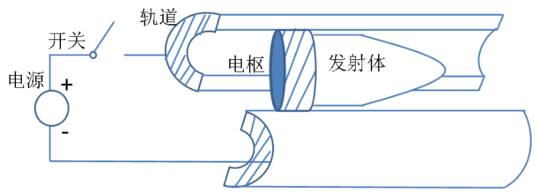
Problem 2

As shown in the figure, a rail-type electromagnetic launcher consists of two parallel fixed long straight rigid metal rails, a high-power electric source, an armature with good sliding electric contact, and the projectile. The current from the source flows through one rail, the armature and the other rail and forms a closed loop, which produces a magnetic field in space. The current-carrying armature is accelerated by the Ampère force and pushes the projectile forward. The mass of the armature is m_s and the mass of the projectile is m_a. For every unit length of the advance of the armature, the resistance and the inductance of the whole loop increase by R_r' and L_r'; the resistance and the inductance introduced by the armature are R_s and L_s; the resistance and the inductance introduced by the connecting wires of the loop are R_0 and L_0. Friction between the rails and the armature, and air resistance, are negligible.

(1) Draw the equivalent circuit diagram of the rail-type electromagnetic launcher, and give the loop (circuit) equation.

(2) Find the relation between the acceleration of the projectile on the rails and the loop current.

(3) Let the loop current be a constant current I (a flat-top pulse current); the total mass of the armature and the projectile is $m_s + m_a = 0.50$ kg, the rail length is $x_{sm} = 500$ m, the increase of the inductance per unit length of rail is $L'_r = 1.0$ μH/m. The projectile starts from rest, and the exit speed is $v_{sm} = 3.0 \times 10^3$ m/s. Find the loop current I and the acceleration time τ.



Labels: 开关 = *switch;* 电源 = *source;* 轨道 = *rail;* 电枢 = *armature;* 发射体 = *projectile.*

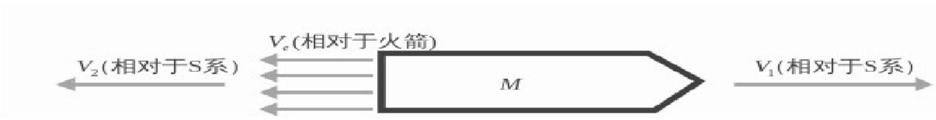
Problem 3

The Russian rocket expert Tsiolkovsky simplified the process of a rocket launch into a model and obtained the earliest ideal rocket equation, which gave the theoretical base for modern rocket and missile engineering. The simplified model is: the rocket, ready for launch, is at rest at some point of an inertial reference frame S; all external forces on the rocket, such as the Earth's gravity, are neglected; the initial rest mass of the rocket (fuel included) is M_i. At the moment $t = 0$ the rocket is ignited and expels gas to the left; the speed of the gas relative to the rocket is constant, equal to V_e, and the rocket is launched to the right. Observed in the frame S, the velocity of the rocket is V_1 and the velocity of the expelled gas is V_2, as shown in the figure. By this model, after the rocket has moved for some time, when its rest mass has changed from M_i to M:

(1) use Newtonian mechanics to derive the relation between the velocity V_1 of the rocket and the mass ratio M/M_i;

(2) use the dynamics of special relativity to derive the relation between the velocity V_1 of the rocket and the mass ratio M/M_i;

(3) when V_1 is much smaller than the speed of light c in vacuum, calculate the difference of the two results above, keeping terms up to the order $(V_1/c)^2$.



Labels: V_e *(*相对于火箭*)* = V_e *(relative to the rocket);* V_1, V_2 *(*相对于S系*)* = V_1, V_2 *(relative to the frame S).*

Problem 4

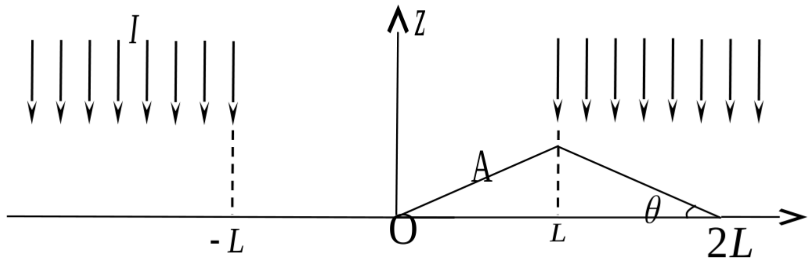
When light is reflected or absorbed at the surface of a body, the photons give their momentum to the body and produce the radiation pressure of light. With radiation pressure one can manipulate small particles precisely (optical tweezers). Suppose that in the region $|y| \geq L$ there is a uniform strong laser field that falls along the negative z direction; its intensity (the light energy that passes through a unit cross-section in unit time) is I. In the region $y \in (-L, L)$ there is no light field. The cross-section is shown in the figure. A small prism-shaped body of density ρ (its cross-section is an isosceles triangle with base $2L$ and base angle θ) lies on the smooth horizontal xy plane; its two upper faces A and B carry a special reflecting layer that reflects the incident light completely. The body is initially at rest, in the position shown in the figure.

(1) Assume that the angle of reflection of a photon equals the angle of incidence, and that the frequency of a photon does not change in the reflection. Find:

  (a) the dynamical equation of the body after the laser field begins to shine on it;

  (b) the time the body needs to move, from its initial position, a distance $3L/2$ in the negative y direction.

(2) If the influence of the motion of the body on the frequency of the reflected photons is taken into account, the frequency of the reflected photons changes slightly. In the process in which the body moves from its initial position through the distance $3L/2$ in the negative y direction, compare qualitatively the frequency of the reflected photons with the frequency of the incident photons, and describe its change with time.



Problem 5

As shown in the figure, a glass cylinder with inner radius R_1, fixed at its upper end, has its bottom below the water surface of a large vessel; it does not touch the bottom of the vessel. A second cylinder of the same glass, with outer radius R_2 (R_2 slightly smaller than R_1), also fixed at its upper end, is placed inside the first one; it does not touch the bottom of the vessel either. The axes of the two glass cylinders coincide and are vertical. The contact angle between water and glass is θ (the angle between the tangent of the gas–liquid interface at its highest point and the solid–liquid interface; see the enlarged view in the figure). The density of water and the surface-tension coefficient of water are ρ and α; the magnitude of the gravitational acceleration at the ground is g.

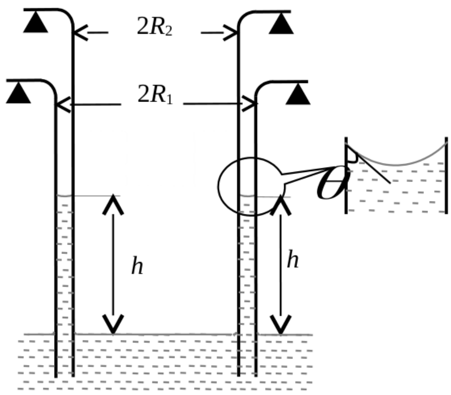
(1) In capillary phenomena, the change of the potential energy of the system caused by the capillary action can be treated, equivalently, as the work done by the surface tension that acts on the solid–liquid–gas contact line while the liquid surface rises or falls. Prove: with the water surface of the large vessel as the zero point of the potential energy of the system, when the water between the two cylinders has risen by the height h because of the capillary action, the capillary potential energy of the system is

$$E_s(h) = -2\pi\alpha\cos\theta\,(R_1 + R_2)\,h;$$

(2) Derive the heat that the system releases in the process in which the water rises because of the capillary action.

(3) If this experiment is made, for a short time, in an airplane at an altitude above ten kilometres (the airplane can be treated as moving uniformly along a straight line), the heat released by the system in the same process is measured to be Q. Estimate the height of the airplane above the ground. Assume that the surface-tension coefficient and the contact angle of the water in the airplane are the same as at the ground.

The calculations above do not take the spin of the Earth into account.



Problem 6

The directions of the orbital revolution of the planets of the solar system are essentially the same. The position of a planet seen by an observer on the Earth is in fact the projection of the planet on the distant background of stars. Because the orbital speeds of the planets and their positions on their orbits differ, the direction in which the observer on the Earth sees a planet move against the background of stars is not always the same as the direction of the orbital motion of the Earth. When the seen direction of the motion of a planet is the same as the direction of the orbital motion of the Earth, the apparent motion is called prograde; when it is opposite, the apparent motion is called retrograde. At the turning point, where the apparent motion changes from prograde to retrograde or from retrograde to prograde, the planet seems to stand still in the sky; this position is called a station. In astronomy, when an outer planet comes to the straight line through the Sun and the Earth, so that the Sun and the outer planet are on the two sides of the Earth, the condition is called an opposition of the planet. The opposition of Mars is a common astronomical phenomenon. Assume that the Earth and Mars both move around the Sun in circles in the same plane, and consider only the attraction of the Sun on the planets. The orbital radius of Mars, R_m, is 1.524 times the orbital radius R_0 of the Earth. Neglect the spin of the Earth. Calculate the time intervals, within the time between two neighbouring oppositions of Mars, during which the apparent motion of Mars is retrograde and during which it is prograde.

Problem 7

The principle of the time-of-flight mass spectrometer (TOFMS) is shown in figure 1. The instrument has three main parts: the ion-source region, the drift region and the detector. Positive ions form in the ion source and are accelerated by the field E_s; they pass the drift region (vacuum, no field) and reach the ion detector. The acceleration distance of an ion in the ion-source region is S, and the drift distance in the drift region is L. Through a record of the arrival time of the ions at the detector, ions can be separated by the size of the mass-to-charge ratio m/q, where m and q are the mass and the charge of an ion. The resolution is the most important property of a TOFMS; this problem discusses or calculates it in different situations. Neglect gravity.

(1) In an ideal TOFMS, different ions are all created at the same position on the X axis in the ion source, at the same moment $t = 0$, with zero initial velocity. The detector measures the moment t at which an ion arrives; its minimum resolvable time is Δt (the error of the measured moment t). Define the resolution of the instrument as $R = m/\Delta m$, where Δm is the minimum resolvable mass that corresponds to the minimum resolvable time Δt. In this case R is completely determined by Δt; derive the relation between R and t, Δt.

(2) In reality the creation positions of the ions differ slightly, by ΔS ($\Delta S \ll S$); therefore ions with the same mass-to-charge ratio do not arrive at the detector at the same time, and the resolution becomes worse. As shown in figure 2, a second accelerating field E_1, of region length D_1, is added after the ion source. Through a suitable choice of the length L of the drift region, the ions with the same m/q, created at the same moment at different positions, can arrive at the detector almost at the same time, so that the effect of ΔS on the resolution becomes smallest. Find this value of L.

(3) To lower the effect of the spread of the creation positions on the resolution further, the reflection-type TOFMS of figure 3 is usually used. Here, on top of the two-stage accelerating fields (E_s and E_1), a reflector is added; it consists of the two-stage fields E_2 and E_3 (region lengths D_2 and D_3), which reverse the flight direction of the ions, so that the effect of ΔS on the resolution becomes smallest; find the value of L in this case. To simplify the calculation, assume the ions move along straight lines parallel to the X direction. (The parameters of the device satisfy $E_sS + E_1D_1 \leq E_2D_2 + E_3D_3$, so that the reversal of the flight direction happens for all ions.)

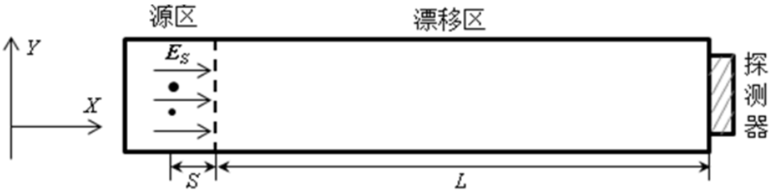


图1

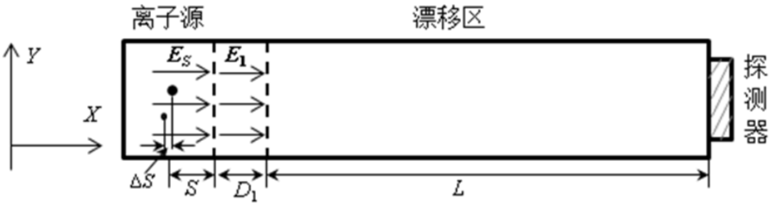
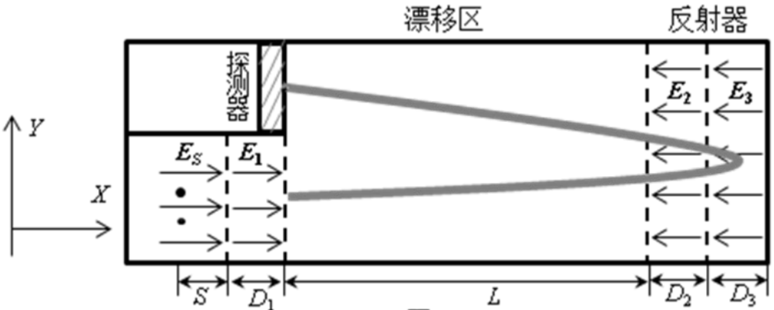


图2



Labels: 离子源(区) *= ion source (region);* 漂移区 *= drift region;* 探测器 *= detector;* 反射器 *= reflector;* 图 *1/2/3 = figure 1/2/3.*

Problem 8

The design of a laser aiming system must take the change of the refractive index of air into account. Because of the conditions of the ground surface, the altitude, the air temperature, the humidity, the air density and other factors, the refractive index of air is not uniform in the atmosphere, and the propagation path of laser light is not a straight line. For simplicity, assume that at some place the refractive index of the air changes with the height y as

$$n^2 = n_0^2 + \alpha^2 y,$$

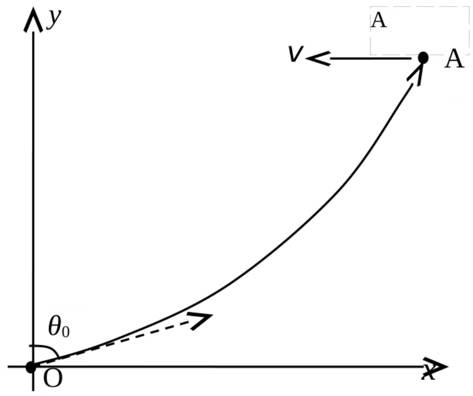
where n_0 is the refractive index of the air at $y = 0$ (the ground), and n_0 and α are known constants greater than zero. The propagation time of the laser light itself is negligible. The laser device is at the coordinate origin O, as shown in the figure.

(1) If the laser beam leaves the device at the angle θ_0 ($0 \le \theta_0 \le \pi/2$) to the vertical y axis, find the equation that describes the propagation path of the laser light.

(2) Assume the target A is in the first quadrant. When the height of the target A is y_a, find the largest x coordinate, $x_{a\,\max}$, of a target A that the laser device can illuminate.

(3) The destruction of a target needs a certain time of illumination. Suppose the target A is inside the attack range of the laser device; its initial position is (x_0, y_0), and it approaches the laser device at the same height with the constant speed v. How must the emission angle θ_0 of the laser change with the time t so that the laser illuminates the target all the time?

(4) The attack of the laser device usually follows the principle of safe destruction: the target aircraft A must be destroyed, and at the same time none of the bombs that the target aircraft A releases horizontally (a bomb is at rest relative to A at the moment of release) may hit the laser device. Assume that the laser device begins to illuminate A with the laser as soon as A enters the attack range. The target A flies horizontally at the height y_a; the destruction of A needs the continuous illumination time t_a; and the laser device at the origin O can destroy it safely. Find the allowed range of the speed of A. Air resistance is negligible; the magnitude of the gravitational acceleration is g.


y
A
v
A
θ0
O
x