The 38th Chinese Physics Olympiad

Final — Theoretical Examination (Problems)

2021

Five problems. Figures are reproduced from the Chinese original; a short glossary of the Chinese labels follows each figure.

Problem 1

A small circular hole in an opaque thin sheet is the region between the black parts in Fig. 1a; its radius is OM $= 1.00\,\text{mm}$. A He–Ne laser of wavelength $\lambda = 632.8\,\text{nm}$ is used as the source; it enters from the left of the hole as a parallel beam at normal incidence. Take a point P on the symmetry axis perpendicular to the hole, on the right-hand side. Relative to P the wave surface at the hole can be regarded as a combination of half-wave zones: with P as centre, and with r_0 the distance from P to the centre O of the hole ($r_0 \gg \lambda$), draw spheres of radii $r_0 + \dfrac{\lambda}{2}$, $r_0 + 2 \times \dfrac{\lambda}{2}$, $r_0 + 3 \times \dfrac{\lambda}{2}$, ... These divide the wave surface in the plane of the hole into N annular zones (N a natural number); the distance from P to the rim M of the hole is $r_0 + N\dfrac{\lambda}{2}$, and the zone of smallest radius is a disc. The zones obtained in this way are called half-wave zones (because the optical path difference from the corresponding edges of two adjacent zones to P is $\lambda/2$). Clearly the number of zones N fixes the position of P. Whenever the existing half-wave zones have to be re-divided or merged, consider only re-dividing each existing half-wave zone into several new half-wave zones, or merging several existing half-wave zones into one new half-wave zone.

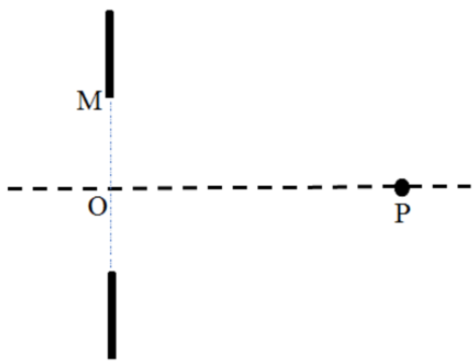

图 1a

Labels: 图 1a = *Fig. 1a.*

(1) If $N = 2n + 1$, find the positions of P corresponding to $n = 0$ and $n = 1$: the point P_0 (P_0 is the rightmost bright point on the axis, called the *principal focus*) and the point P_1 (P_1 is also a bright point, called a *secondary focus*).

(2) If $N = 4$ (a 4-zone plate) and transparent material is placed on the 1st and 3rd half-wave zones (the grey parts in Fig. 1b), so that the light passing through this transparent material gains an extra optical path $\dfrac{\lambda}{2}$, find

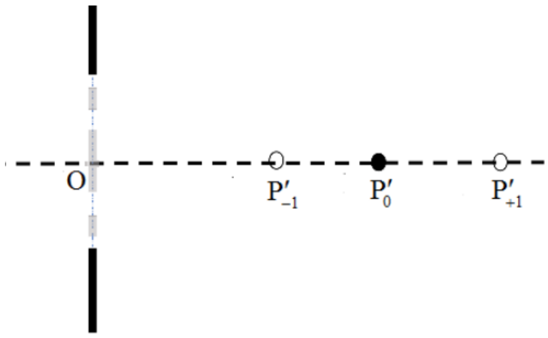

图 1b

Labels: 图 1b = *Fig. 1b.*

(i) the position of the principal focus P'_0 of this 4-zone plate;

(ii) the position of the dark point P'_{-1} immediately to the left of the principal focus P'_0;

(iii) the position of the dark point P'_{+1} immediately to the right of the principal focus P'_0.

(3) A zone plate can not only focus parallel light, it can also form images. The focusing of parallel light by the 4-zone plate above corresponds to the case of infinite object distance with image distance equal to the focal length. Place a point source (the object) at the point S on the axis, 3 m to the left of O; its image point is S′, as shown in Fig. 1c.

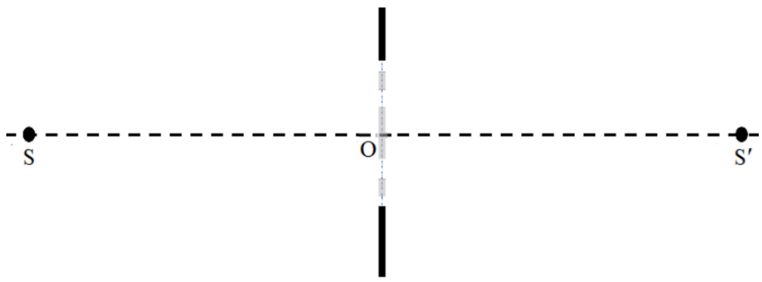

图 1c

Labels: 图 1c = *Fig. 1c.*

(i) What is the image distance OS′ that corresponds to the principal focus of the 4-zone plate? Verify whether the imaging formula is satisfied.

(ii) If the image formed by this zone plate is not unique and there are other image points on the axis, where is the image point closest to the one obtained in (3)(i)? What is the focal length of the secondary focus corresponding to this imaging process? (The imaging formula may not be used.)

(iii) If the object is placed to the left of this zone plate at the distance $OP'_0/2$ from O, find the type (virtual or real) and the position of the images formed when the principal focus of the 4-zone plate and the secondary focus described in (3)(ii) are used as the focus. (The imaging formula may not be used.)

Problem 2

In June 2021 the Shenzhou-12 manned rocket docked successfully with the Tiangong space station. This involves the problem of a chaser (the Shenzhou-12 rocket) meeting a target (the space station) on an orbit around the Earth. This problem uses a Hohmann transfer scheme to study how the chaser changes its velocity (speed and direction) so as to dock with (meet) the target on its fixed orbit.

As shown in Fig. 2a, the target A and the chaser c both move counter-clockwise with speed v_0 on a circular orbit of radius r_0. At time 0 their positions are

$$\theta_{A,i} = \theta_0, \quad \theta_{c,i} = 0, \quad r_{A,i} = r_{c,i} = r_0.$$

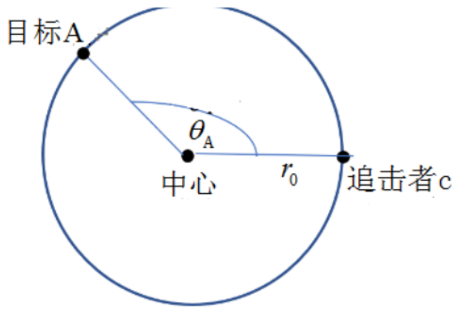

图 2a.

Labels: 目标A = *target A;* 中心 = *centre;* 追击者c = *chaser c;* 图 2a = *Fig. 2a.*

At this instant the chaser c fires instantaneously and its velocity changes instantaneously by $\Delta \boldsymbol{v}$ (as shown in Fig. 2b); the orbit of c also changes instantaneously from the circle of radius r_0 to the ellipse shown in Fig. 2c. The angle between the major axis of the elliptical orbit and the polar-axis direction (the line from the centre to the position of c at the moment of firing) is ϕ (ϕ is measured clockwise). The angle between the direction of motion of c and the polar-axis direction is denoted θ_c (the positive sense of θ_c is counter-clockwise), and the distance of c from the centre is $r_c(\theta_c)$.

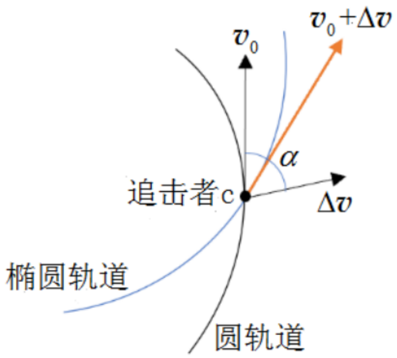

图 2b

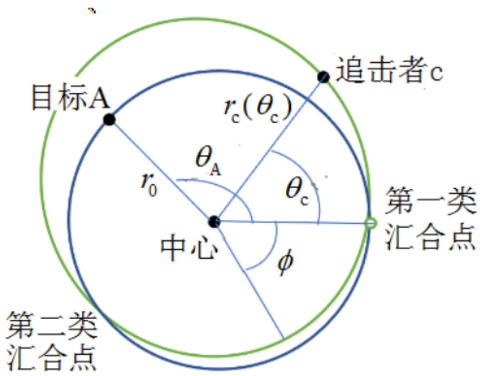

图 2c

Labels (Fig. 2b): 追击者c = *chaser c;* 椭圆轨道 = *elliptical orbit;* 圆轨道 = *circular orbit. Labels (Fig. 2c):* 目标A = *target A;* 中心 = *centre;* 追击者c = *chaser c;* 第一类汇合点 = *junction point of the first kind;* 第二类汇合点 = *junction point of the second kind.*

(1) If the mass m, the energy E (in fact the total mechanical energy of the system formed by the flying body and the Earth) and the angular momentum L of the flying body are all known, express the orbital parameters R and ε in terms of E, L, m and the given quantities r_0, v_0, etc. Given: an unrotated elliptical orbit (major axis along the polar-axis direction) has, in polar coordinates with the origin at the right focus, the form

$$r(\theta) = \frac{R}{1 + \varepsilon \cos\theta},$$

where R is the orbit-size parameter and ε is the orbital eccentricity; together they are called the orbital parameters.

(2) Write the expression for the orbit $r_{\mathrm{c}}(\theta_{\mathrm{c}})$ of the chaser c after firing (see Fig. 2c), in terms of r_0, the eccentricity ε and ϕ.

(3) Write the ratio $T_{\mathrm{c}}/T_{\mathrm{A}}$ of the orbital period T_{c} of the chaser c after firing to the period T_{A} of the target A, in terms of ε and ϕ.

(4) Define two firing parameters (see Fig. 2b): the dimensionless rate of velocity change $\delta = \left|\dfrac{\Delta \boldsymbol{v}}{v_0}\right|$, and the angle α between $\Delta \boldsymbol{v}$ and $\boldsymbol{v}_0$ ($\alpha = 0$ when they coincide, the clockwise sense being positive). Express the eccentricity ε of the orbit of the chaser c, and $\varepsilon\cos\phi$, in terms of the firing parameters δ and α.

(5) Consider the case in which the chaser c and the target A meet at a junction point of the first kind (see Fig. 2c). Let n_{A} be the number of times the target A passes the junction point of the first kind counted from time 0, and n_{c} the number of times the chaser c passes the junction point of the first kind (the initial instant not counted). At time 0, $\theta_{\mathrm{A},i} = \theta_0$, $\theta_{\mathrm{c},i} = 0$. Find n_{A} in terms of n_{c}, θ_0, ε and ϕ.

(6) Express n_{A} in terms of δ and α. With δ fixed, find the two simple, obvious extremum points α_0 of the function $n_{\mathrm{A}}(\alpha)$ with respect to α (so that even if the firing angle α deviates a little from α_0 the solution still holds approximately, which makes a successful docking easier).

(7) If one of the two values of α_0 above is taken,

(i) there is an upper limit $\delta_{\max}$ for δ (that is, if $\delta > \delta_{\max}$ the chaser c and the target A will not meet); find $\delta_{\max}$;

(ii) let the initial value of θ_{A} be θ_0; write the relation between δ and θ_0, n_{A}, n_{c}, and find the value of δ when $\theta_0 = \dfrac{\pi}{2}$, $n_{\mathrm{A}} = 2$, $n_{\mathrm{c}} = 1$.

Problem 3

About one quarter of the matter in the universe may exist in the form of dark matter. Dark matter may be made of a new kind of elementary particle. People have long been looking for the interaction between dark-matter particles and known particles; one important class of experiments searches for the interaction between dark-matter particles and atomic nuclei. Such experiments usually take some material as the target; when a dark-matter particle flies into the detector it collides with a nucleus of the target material. In this process the originally almost stationary nucleus receives part of the kinetic energy of the dark-matter particle, moves in the target material and produces light signals and electric signals. The dark-matter detector can detect these signals and thus observe dark matter.

(1) Take as the target material the xenon isotope nucleus $^{132}\mathrm{Xe}$. Assume that the masses of the dark-matter particle X and of the $^{132}\mathrm{Xe}$ nucleus are both 132 times the proton mass m_{p}; in the centre-of-mass frame of the particle X and the $^{132}\mathrm{Xe}$ nucleus the scattering is isotropic (the outgoing particle appears with equal probability in every direction). Suppose that a dark-matter particle X enters the detector along the z direction with a speed of magnitude 200 km/s and collides with a $^{132}\mathrm{Xe}$ target nucleus at rest. Derive, for the laboratory frame (the frame in which the target nucleus $^{132}\mathrm{Xe}$ was at rest before the collision), after the collision:

(i) the relation between the momentum of the $^{132}\mathrm{Xe}$ nucleus and its outgoing angle;

(ii) the angular distribution of the ^{132}Xe nucleus (its probability distribution over the outgoing angle);

(iii) the distribution of the kinetic energy of the ^{132}Xe nucleus.

(2) The scattering cross-section is often used to express the strength of the interaction between two particles. A scattering cross-section is like a certain cross-sectional area, and this cross-sectional area may be understood as the effective area for a collision between two particles. For example, the cross-sectional area for a collision between two rigid balls of radii r_1 and r_2 is $\pi(r_1+r_2)^2$, which is the scattering cross-section of the two balls. Now the detector contains 1.0 t of xenon atoms; the scattering cross-section for a collision between a dark-matter particle X and a ^{132}Xe nucleus is $1.0 \times 10^{-38}\,\mathrm{cm}^2$; assume the speeds of all dark-matter particles X are 200 km/s; assume the density of dark matter near the Earth is $0.40\, m_\mathrm{p}/\mathrm{cm}^3$. How many collisions between dark-matter particles X and ^{132}Xe nuclei can take place in this detector per year? Avogadro's number is $N_\mathrm{A} = 6.023 \times 10^{23}$. For simplicity assume the proton rest mass is 1.0 g/mol.

(3) The signal detected by the detector is produced by the kinetic energy that the ^{132}Xe nucleus acquires in the collision. The signal can be recorded only if this kinetic energy exceeds the threshold of the detector. Assume this threshold is 10 keV. Estimate how many collisions between dark-matter particles X and ^{132}Xe nuclei this detector can detect per year. The proton rest mass is 0.94 GeV/c^2, where c is the speed of light in vacuum.

(4) The speeds of the dark-matter particles in the Milky Way obey a certain distribution $f(v)$; the speed of a dark-matter particle has an upper limit v_max, that is, $f(v) = 0$ when $v > v_\mathrm{max}$. Answer: what causes this upper limit?

(5) For a laboratory frame on the Earth, the upper limit of the speed distribution of the dark-matter particles is 0.002 times the speed of light in vacuum. Moreover we do not in fact know what the mass of a dark-matter particle is. Below what mass of the dark-matter particle would the above detector be unable to detect a dark-matter signal?

Problem 4

Electric interactions play a very important role in living organisms. When a DNA-like acidic molecule is put into water, some loosely attached atoms of the molecule may dissociate; the positive ions drift away and leave some electrons behind, so that this macromolecule becomes negatively charged. Similarly, cell membranes in water also become negatively charged in this way, and the electrostatic repulsion between them keeps such macromolecules or cells from "clumping". Since the substance itself is electrically neutral, the large number of positive ions floating in the solution makes the electrostatic force between these macromolecules decay with distance. At the normal temperature T consider the following questions.

(1) (i) Consider the region just outside the outer side of a negatively charged cell membrane (the side facing the region in which the positive ions float), close to its surface. The cell membrane may then be regarded as an infinite plane. Take the x axis perpendicular to the outer membrane surface, pointing outwards, with the origin on the outer membrane surface. The permittivity of the solution around the membrane is ε; the surface charge density uniformly distributed on the outer membrane surface is $-\sigma$ ($\sigma > 0$). The system stays in thermal equilibrium with the environment. Because of the electrostatic interaction the density of the positive ions in the solution is not uniform, but every local part may be regarded as being in equilibrium. The behaviour of the positive ions is then similar to that of the air molecules above the ground, except that here the positive ions are acted on by the electrostatic field while the air molecules are acted on by the

gravitational field. Take the number density n_0 of the positive ions immediately at the outer membrane surface as an undetermined constant. Write the differential equation satisfied by the potential φ near the outer membrane surface. The electron charge is $-e$, the charge of a positive ion is $+e$, Boltzmann's constant is k; neglect gravity and the buoyancy of water. Hint: Boltzmann statistics says that the probability that a particle is in a state of energy E is proportional to $\exp\left(-\dfrac{E}{kT}\right)$, T being the absolute temperature of the equilibrium state of the particle.

(ii) The liquid tissue inside the cell-membrane surface may be regarded as conducting; take the potential on the membrane surface to be zero. Write the boundary conditions satisfied by the potential φ on the outer membrane surface.

(iii) With the given boundary conditions, take the logarithmic function $\varphi(x) = A\ln(1 + Bx)$ as a trial solution of the differential equation obtained in (1)(i). Determine the values of the constants n_0, A, B, and express $\varphi(x)$ and $n(x)$ (the number density of the positive ions at x) in terms of σ, ε, T, e, k.

(iv) Verify that the $n(x)$ obtained guarantees that the whole system is electrically neutral.

(v) Find the electrostatic field energy U_f in a sufficiently tall cylindrical volume whose base is a unit area of the charged outer membrane surface, and also the total electrostatic potential energy U_e of all the positive ions in the same volume in the electrostatic field.

(2) Consider the case of two cells close to each other. The system formed by the outer sides of the two cell membranes and by the region between them may be studied with the following simplified model: two uniformly negatively charged infinite parallel planes, each with surface charge density $-\sigma$, a distance $2D$ apart. The midpoint between the two plates is chosen as the origin, the x axis is perpendicular to the plates, the potential at the origin is chosen to be zero, and the number density n_0 of the positive ions at the origin is taken as an undetermined parameter. Take the function $\varphi(x) = A\ln[\cos(Bx)]$ as a trial solution of the differential equation.

(i) From the differential equation satisfied by $\varphi(x)$, determine the constants A, B in terms of T, ε, e, k, n_0.

(ii) Find n_0 in terms of D, σ, ε, T, e, k, θ (if a transcendental equation is met it need not be solved; denote by θ its solution in the physical region).

(iii) Give the difference $\Delta n \equiv n(\pm D) - n_0$ between the particle number density of the positive ions at the outer cell-membrane surface and that at the centre between the two membranes, in terms of σ, ε, T, k.

(iv) Verify that the $n(x)$ obtained guarantees that the whole system is electrically neutral.

(v) Give the resultant force per unit area on a charged plane (a cell membrane). Both inside and outside the cell membrane there is water of the same density. Hint: the behaviour of the positive ions in water is similar to that of an ideal gas, and each local region is in thermal equilibrium.

(vi) If the other conditions are unchanged and the two plates may still be regarded as infinite, discuss how the magnitude of the resultant force per unit area on a charged plane changes with the distance D in the two different limiting cases of small separation and large separation.

Problem 5

(1) The spin magnetic moment of an electron is

$$\boldsymbol{\mu} = \frac{-e}{m}\boldsymbol{S},$$

where $-e$ ($e > 0$) is the electron charge, m is the electron mass, $\dfrac{-e}{m}$ is the charge-to-mass ratio of the electron; $\boldsymbol{S}$ is the spin angular momentum, whose z component S_z can take only two values: $S_z = \dfrac{1}{2}\hbar$ for spin up and $S_z = -\dfrac{1}{2}\hbar$ for spin down, where $\hbar = \dfrac{h}{2\pi}$ and h is Planck's constant. Assume that the magnetic moment of a molecule or an atom equals the spin magnetic moment of one electron.

(i) When a paramagnet is placed in an external magnetic field it becomes magnetized, because the probabilities that the magnetic moments of its molecules are aligned parallel or antiparallel to the field direction change. The degree of magnetization is described by the magnetic moment per unit volume (the magnetization) $\boldsymbol{M}$, with $\boldsymbol{M} = \chi\boldsymbol{H}$, where χ is the magnetic susceptibility, $\boldsymbol{H} \equiv \dfrac{\boldsymbol{B}}{\mu_0} - \boldsymbol{M}$ is the magnetic field strength, $\boldsymbol{B}$ is the magnetic induction and μ_0 is the vacuum permeability. At an ordinary temperature T (absolute temperature), $\chi \ll 1$ and $kT \gg \dfrac{e\hbar}{2m}B$ (k is Boltzmann's constant); assume the magnetic induction $\boldsymbol{B}$ is uniform and the molecular number density of the paramagnet is n. Use Boltzmann statistics to derive that the susceptibility of the paramagnet satisfies $\chi = \dfrac{C}{T}$, and give the expression for C. Given: Boltzmann statistics says that the probability that a particle is in a certain energy state is proportional to $\mathrm{e}^{-\frac{E}{kT}}$, where E is the energy of that state and T is the absolute temperature of the equilibrium state of the particle.

(ii) A ferromagnet differs from a paramagnet: below the Curie temperature a ferromagnet can develop and keep rather strong magnetism in an environment with no external field; this is called spontaneous magnetization. Spontaneous magnetization produces "domains"; in each "domain" the molecular magnetic moments are all aligned along the same direction (aligned), and the ferromagnet is then in the ferromagnetic phase. The reason why the magnetic moments become aligned is a quantum effect; one may also regard it as a very strong effective magnetic field inside, which Weiss called the "molecular field". The "molecular field" is about three orders of magnitude larger than the field produced by a single spin magnetic moment at the interatomic distance. Above the Curie temperature a ferromagnet turns into a paramagnet, and its susceptibility obeys the Curie–Weiss law $\chi = \dfrac{C}{T-\theta}$ (where θ is the Curie temperature), which differs from that of an ordinary paramagnet; the difference comes from the "molecular field" H_m inside the ferromagnet, and H_m can be written as $H_m = \gamma M$. Assume the molecular number density of the ferromagnet is also n. Give the expression for γ (the Curie temperature θ may be regarded as known); and derive the expression for the molecular field H_m in the case in which all the magnetic moments are aligned.

(2) In a ferromagnetic crystal the atoms are arranged on a periodic lattice. The magnetic field felt by each atom can be understood as the average of the "molecular fields" supplied by the atoms on its nearest-neighbour lattice sites.

(i) Consider the one-dimensional lattice spin system shown in Fig. 5a, in which the magnetic moments on all the sites are aligned. The Curie temperature of this system is θ. Assume that the relation between H_m and θ derived in the previous part still holds, and that this system contains N ($N \gg 1$) lattice sites with one atom on each site; after spontaneous magnetization the atomic spins are arranged parallel, and such an arrangement makes the energy of the system lowest. Give the effective magnetic field produced by one spin at its nearest-neighbour lattice sites, and the magnetic energy of the whole system.

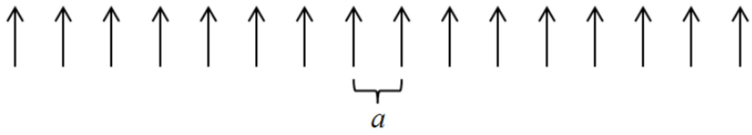

图 5a

Labels: 图 5a = *Fig. 5a.*

(ii) Assume that the spin on one non-end lattice site of this one-dimensional lattice spin system flips and becomes antiparallel, as shown in Fig. 5b. Compared with the case of Fig. 5a, by how much does the energy of the system increase?

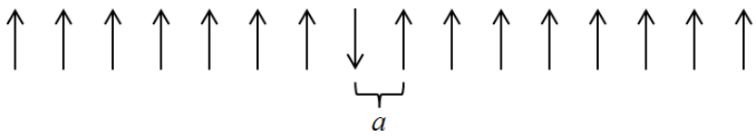

图 5b

Labels: 图 5b = *Fig. 5b.*

(3) If the atomic spins are alternately reversed, the material is called an antiferromagnet. An antiferromagnet has a special lattice structure: this structure consists of an A sublattice and a B sublattice, as shown in Fig. 5c. The nearest neighbours of an A site are all B sites, and only the next-nearest neighbours are A sites; similarly for B sites. The nearest-neighbour "molecular field" of an antiferromagnet has the direction opposite to that of a ferromagnet, and besides the nearest-neighbour spins, the next-nearest-neighbour spins also contribute to the "molecular field". The "molecular fields" at the A and B sites can then be written respectively as

$$\boldsymbol{H}_{\mathrm{mA}} = -\alpha_{\mathrm{AB}}\boldsymbol{\mu}_{\mathrm{B}} - \alpha_{\mathrm{AA}}\boldsymbol{\mu}_{\mathrm{A}}, \qquad \boldsymbol{H}_{\mathrm{mB}} = -\alpha_{\mathrm{AB}}\boldsymbol{\mu}_{\mathrm{A}} - \alpha_{\mathrm{BB}}\boldsymbol{\mu}_{\mathrm{B}},$$

where $\boldsymbol{\mu}_{\mathrm{A}}$, $\boldsymbol{\mu}_{\mathrm{B}}$ are the spin magnetic moments on the A and B sites. Assume all A-site atoms are identical and all B-site atoms are identical, so that $\alpha_{\mathrm{AA}} = \alpha_{\mathrm{BB}} = \alpha$, $\alpha_{\mathrm{AB}} = \alpha_{\mathrm{BA}} = \beta$, $\beta > |\alpha|$.

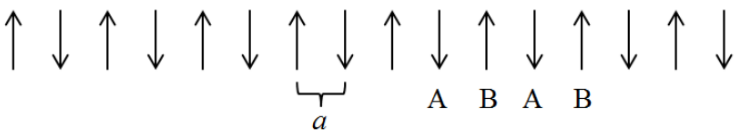

图 5c

Labels: 图 5c = *Fig. 5c.*

(i) Give the magnetic energy of an antiferromagnet with N lattice sites ($N/2$ sites each of type A and B).

(ii) The Néel temperature is the temperature above which the antiferromagnetic phase disappears and the antiferromagnet becomes paramagnetic. At this temperature the magnetizations and the fields of the A sublattice and of the B sublattice satisfy respectively

$$M_{\mathrm{A}} = \frac{C}{2T} H_{\mathrm{A}}, \qquad M_{\mathrm{B}} = \frac{C}{2T} H_{\mathrm{B}},$$

where the coefficient C is analogous to that in question (1)(i). Give the Néel temperature of a one-dimensional antiferromagnet.

(4) Now consider another situation: the atoms are distributed on a square lattice with lattice constant a in a two-dimensional x–y plane, and the magnetic moments of the atoms can rotate within the plane. As shown in Fig. 5d, in a planar region of size L ($L \gg a$) the distribution of the magnetic moments is shown by the arrows at the various points. Because of the "molecular field" effect there is a rather strong interaction between the magnetic moments, and this interaction is appreciable only between nearest neighbours. Let the angle between the spin angular momentum $\boldsymbol{S}_i$ on the i-th lattice site of this plane and the x axis be $\theta_i \in [0, 2\pi]$. The magnetic energy of the system can be written as

$$E(\{\theta_i\}) = -J' \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j = -J \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j),$$

where J', J (J', $J > 0$) are constants, $\langle i,j \rangle$ denotes all nearest-neighbour lattice sites i and j (a lattice site generally has four nearest-neighbour sites), and the configuration of the system is given by $\{\theta_i\}$. Assume that the system reaches thermal equilibrium with the environment and that the volume of the system does not change; the Helmholtz free energy is then $F = E - TS$, where E is the internal energy of the system and S is the entropy of the system.

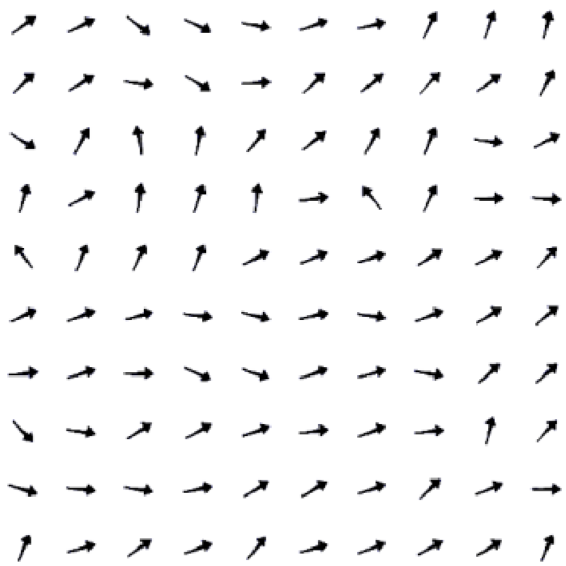

图 5d

Labels: 图 5d = *Fig. 5d.*

(i) A vortex configuration that may exist in the system is shown in Fig. 5e. Going once round a closed loop that encircles the vortex centre, the sum of the differences of the orientation angles of the magnetic moments on nearest-neighbour sites $\langle i,j\rangle$ on this closed loop is

$$\sum_{\langle i,j\rangle}(\theta_i-\theta_j)=2\pi l.$$

Here l is a positive integer related to the vortex structure; the case $l=1$ is shown in Fig. 5e. For convenience, consider a closed loop (a circle) centred on the vortex centre with a rather large radius r; find the average value $\langle\theta_i-\theta_j\rangle$ of the differences of the orientation angles of neighbouring lattice sites on the circle.

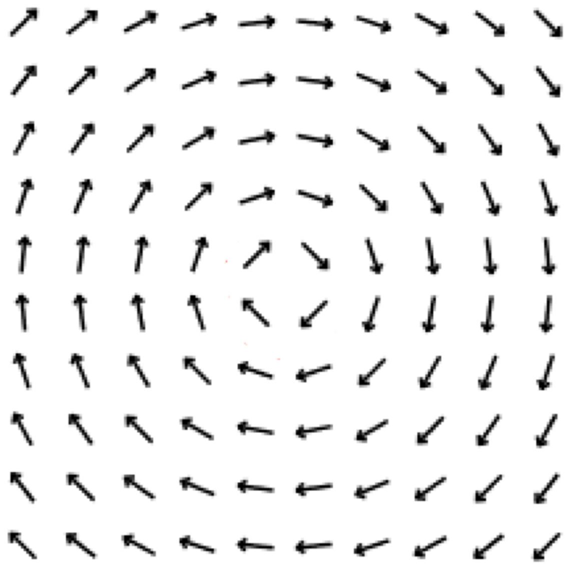

图 5e

Labels: 图 5e = *Fig. 5e.*

(ii) When the radius r of the loop around the vortex centre is rather large, one may approximately regard the magnetic moments as arranged in turn on a series of concentric circles whose radii differ by a, and the angle between nearest-neighbour magnetic moments on adjacent concentric circles may be taken as zero. The same approximation is made in the region of small radius; this has little effect on the total energy. On this basis estimate the energy of one vortex system such as that shown in Fig. 5e.

(iii) If the centre of the vortex is placed at different positions, the system may be regarded as being in different states. Assuming that there is only one vortex on a square lattice of size L and lattice constant a, estimate the number Ω of possible states, and then compute the entropy $S\equiv k\ln\Omega$ of the system.

(iv) If there exists a temperature such that the properties of the system differ markedly above and below it, then a phase transition may occur. For a vortex system, find the possible phase-transition temperature and explain it.