The 41st Chinese Physics Olympiad

Final — Theoretical Examination (Problems)

26 October 2024, 9:00–12:00

Six problems, 320 marks in total. The source document contains the problems only; no official solutions are included. Figures are reproduced from the Chinese original, with a short glossary of the Chinese labels after each figure.

Problem 1 (50 points)

Experiments have found that some atomic nuclei emit an electron and turn into a nucleus of another kind, and that the kinetic energy of the emitted electron has a continuous spectrum. This is called β^- decay of the nucleus. For example, the kinetic-energy spectrum of the electrons emitted in the β^- decay of the nucleus ${}^{210}_{83}\mathrm{Bi}$ (measured in the rest frame of ${}^{210}_{83}\mathrm{Bi}$) is shown in Fig. 1a. The rest mass of the electron is $m_e = 0.511\,\mathrm{MeV}/c^2$. Neglect the binding energy of the electrons in the atom.

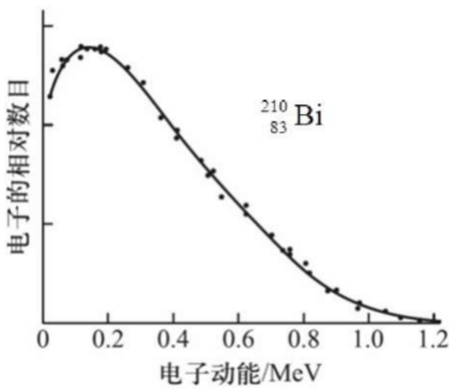


图 1a

Labels: 电子的相对数目 = *relative number of electrons;* 电子动能/MeV = *electron kinetic energy / MeV;* 图 1a = *Fig. 1a.*

(1) Let the rest mass of the parent atom that undergoes β^- decay be $M_\mathrm{P}(Z, A)$ and the rest mass of the daughter atom produced by the β^- decay be $M_\mathrm{D}(Z+1, A)$, where Z and A are the atomic number and the nucleon number of the parent atom. Because it is very hard to strip all the atomic electrons in order to study the β^- decay of the bare nucleus, the β^- decay of a nucleus is usually studied at the level of the atom. Give the condition for an atom to undergo β^- decay.

(2) The atomic mass of ${}^{210}_{83}\mathrm{Bi}$ is $M_\mathrm{P} = 209.984130$ amu (here amu is the atomic mass unit, 1 amu = 931.494 MeV/c^2), and the atomic mass produced after the β^- decay is $M_\mathrm{D} = 209.982883$ amu. Suppose that, apart from the emitted β^- electron, no other new particle is produced in the β^- decay. What is then the kinetic energy of the β^- electron (in MeV)? Assume the ${}^{210}_{83}\mathrm{Bi}$ atom is initially at rest.

(3) The kinetic energy of the β^- electron obtained in part (2) is completely determined; this clearly disagrees with the continuous spectrum observed in experiment. Hence some other

particle must be produced in the β^- decay. Assuming that exactly one other particle is produced, estimate the range of its rest mass from the experimental result shown in Fig. 1a.

(4) Research shows that the elementary process of the β^- decay of a nucleus is the conversion of one of its neutrons n into an electron e^-, a proton p and one other particle X. The neutron, the electron and the proton each carry an intrinsic angular momentum, called the spin of the particle. Further research shows that the spins of the neutron, the electron and the proton are all quantized: the magnitude of the projection of each of them on any given direction is $\frac{1}{2}\hbar$ (with $\hbar = \frac{h}{2\pi}$, h being Planck's constant). Let microscopic particle 1 have angular momentum $\boldsymbol{L}_1$ (its largest projection on any given direction is $l_1\hbar$) and microscopic particle 2 have angular momentum $\boldsymbol{L}_2$ (its largest projection on any given direction is $l_2\hbar$). The possible values of the largest projection, on any given direction, of the total angular momentum $\boldsymbol{L} = \boldsymbol{L}_1 + \boldsymbol{L}_2$ of the two particles are

$$|l_1 - l_2|\hbar, \quad (|l_1 - l_2| + 1)\hbar, \quad \cdots, \quad (l_1 + l_2)\hbar.$$

Find the electric charge and the spin of the particle X.

Problem 2 (50 points)

The problem of the thermal vibrations of a crystal lattice can often be simplified to a problem of coupled spring oscillators. Consider a model of a one-dimensional coupled spring-oscillator system. There are N ($N \gg 1$) large beads of mass M, numbered $1, 3, 5, \dots, 2N-1$, and N small beads of mass m, numbered $2, 4, 6, \dots, 2N$. These $2N$ beads are threaded, in increasing order of their numbers, on a smooth, horizontal, closed, rigid, fixed large circular ring. Every pair of beads with adjacent numbers (including bead number 1 and bead number $2N$) is connected by an identical light spring of stiffness k, and the system is in static equilibrium.

Now study the small vibrations of the above one-dimensional coupled spring-oscillator system S about its equilibrium state. Since the radius of the ring is much larger than the spacing between adjacent beads, each bead may be regarded as a point mass vibrating along its own straight line.

(1) Write down the equation of motion satisfied by the i-th bead ($i = 1, 2, \dots, 2N$). Let its displacement from the equilibrium position be x_i; the positive direction of every x_i is the counter-clockwise direction of the ring.

(2) S has many forms of vibration. Among them there is a special class of vibrational modes in which all the beads perform steady small vibrations with the same frequency. These are called the eigenmodes of S, and their frequencies are called the eigenfrequencies. Find all the possible eigenfrequencies ω of S. (Hint: you may try to solve the problem using a wave (travelling-wave) form, in which the phase difference between the vibrations of adjacent beads is α.)

(3) For an eigenmode of frequency ω, let the amplitude of every large bead be A and the amplitude of every small bead be B. Find

(3.1) the relation between A and B;

(3.2) the average kinetic energy $\langle E_\mathrm{k}\rangle$ of all the beads over one period of oscillation, and the total energy E of S. Neither result may contain B. (Take the energy of the static equilibrium state to be zero.)

(4) Suppose bead number 1 is locked and cannot move, while all the other beads can still slide without friction. Find all the eigenfrequencies ω of the small vibrations of S in this case; and, for an eigenmode of frequency ω, derive the expressions $x_i = x_i(t)$ ($i = 1, 2, \dots, 2N$). No result may contain B.

(5) Suppose the lock on bead number 1 is released and its mass is changed to μ. Even in this case the system still has some eigenfrequencies ω that do not depend on μ at all. Find all the eigenfrequencies ω of the small vibrations of S in this case that do not depend on μ; and, for an eigenmode of frequency ω, derive the expressions $x_i = x_i(t)$ $(i = 1, 2, \ldots, 2N)$. No result may contain B.

Problem 3 (50 points)

In this problem some physical quantities are written as complex numbers. For example, the dependence of a component of the electric field strength on the propagation distance l and the time t is written in complex form as

$$\tilde{E}(t) = Ee^{i(kl-\omega t+\phi_0)},$$

where ω and k are the angular frequency and the magnitude of the wave vector, and ϕ_0 is the initial phase; $\tilde{E}$ is a complex number, and the corresponding E is the modulus $|\tilde{E}|$ of $\tilde{E}$; and so on.

Part one: matrix representation of optical elements.

(1)

(1.1) A Mach–Zehnder (MZ) interferometer is sketched in Fig. 3a. The incident light passes through a half-transmitting, half-reflecting beam splitter, then through two fully reflecting mirrors (total reflectors), and is thereby divided into an upper path (towards M1) and a lower path (towards M2); it then passes through a second, identical half-transmitting half-reflecting beam splitter, where interference occurs. The optical paths can be adjusted so that the detector below the second beam splitter measures zero intensity. Where has the energy of the incident light gone in this case? Draw a figure and describe the answer briefly.

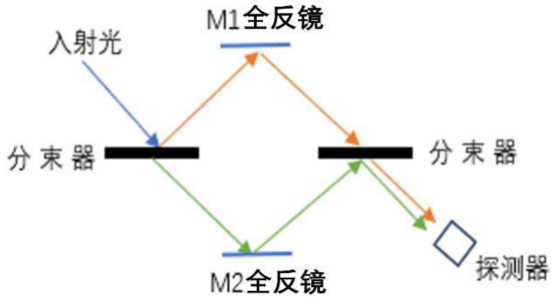


图 3a：马赫-曾德尔干涉仪示意图（部分）

Labels: 入射光 = incident light; 全反镜 = total reflector (fully reflecting mirror); 分束器 = beam splitter; 探测器 = detector; 图 3a：马赫-曾德尔干涉仪示意图（部分） = Fig. 3a: sketch of the Mach–Zehnder interferometer (partial).

(1.2) The set-up of Fig. 3a can be used to study the relation between the transmission coefficient and the reflection coefficient of a beam splitter. Here the beam splitter is a 50/50 half-transmitting half-reflecting element (for all polarizations), i.e. the magnitudes of the reflection coefficient $\tilde{r}$ and of the transmission coefficient $\tilde{t}$ satisfy

$$r = |\tilde{r}| = t = |\tilde{t}| = \frac{\sqrt{2}}{2}.$$

Both reflection and transmission introduce phase shifts, i.e.

$$\tilde{r} = re^{i\delta_r}, \qquad \tilde{t} = te^{i\delta_t}, \qquad \frac{\tilde{t}}{\tilde{r}} = e^{i(\delta_t-\delta_r)} = e^{i\phi}.$$

Find ϕ.

(2) Figure 3b defines the positive directions of the P and S polarizations of light. In Fig. 3b(a), $\boldsymbol{k}$ is the propagation direction of the light, and P–S–$\boldsymbol{k}$ form a right-handed orthogonal triad; the P direction is also called the horizontal direction H, and the S direction is also called the vertical direction V. Figure 3b(b) shows the P, S and $\boldsymbol{k}$ directions of the incident, reflected and transmitted light at a beam splitter. Taking reflection as an example, $\boldsymbol{k}_r$, P_r, S_r denote respectively the propagation direction of the reflected light and the positive directions of the P and S polarizations of the reflected light. After the beam splitter, the P_r and S_r polarization components of the reflected light can both be expressed through the P_i and S_i components of the incident light:

$$\tilde{E}_{r\mathrm{P}} = \tilde{a}\,\tilde{E}_{i\mathrm{P}} + \tilde{b}\,\tilde{E}_{i\mathrm{S}}, \qquad \tilde{E}_{r\mathrm{S}} = \tilde{c}\,\tilde{E}_{i\mathrm{P}} + \tilde{d}\,\tilde{E}_{i\mathrm{S}},$$

and these can be written as a matrix relation

$$\begin{pmatrix} \tilde{E}_{r\mathrm{P}} \\ \tilde{E}_{r\mathrm{S}} \end{pmatrix} = \begin{pmatrix} \tilde{a} & \tilde{b} \\ \tilde{c} & \tilde{d} \end{pmatrix} \begin{pmatrix} \tilde{E}_{i\mathrm{P}} \\ \tilde{E}_{i\mathrm{S}} \end{pmatrix}.$$

The 2×2 matrix in the above expression is called the reflection matrix of the beam splitter, and similarly for the others. The reflection coefficients of the beam splitter for the P and S components are $\tilde{r}_\mathrm{P} = r_\mathrm{P} e^{i\delta_{r\mathrm{P}}}$ and $\tilde{r}_\mathrm{S} = r_\mathrm{S} e^{i\delta_{r\mathrm{S}}}$. Write down the explicit forms of the reflection matrix $\tilde{R}$ and of the transmission matrix $\tilde{T}$ of the beam splitter (in terms of the given quantities of this part and of constants).

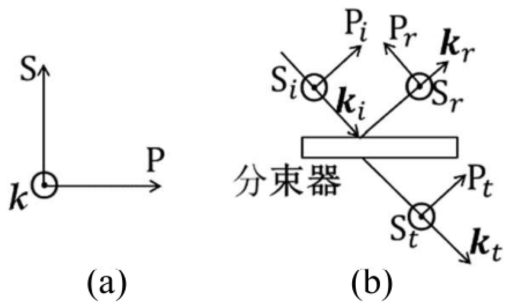


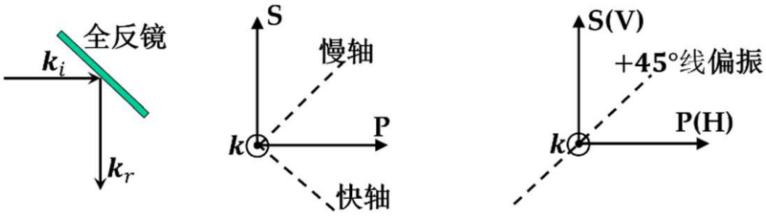
图 3b

Labels: 分束器 *= beam splitter;* 图 3b *= Fig. 3b.*

(3) Figure 3c(a) is a sketch of a total reflector. Write down its reflection matrix $\tilde{M}$.

(4) For a quarter-wave plate, light polarized along its fast axis leads, in phase, the light polarized along its slow axis by $\dfrac{\pi}{2}$ after passing through the plate. Figure 3c(b) is a sketch of a quarter-wave plate whose fast axis is along the $-45°$ direction, with $\boldsymbol{k}$ perpendicular to the plate. Write down the matrix $\tilde{Q}$ of this plate, and express it in the standard form $\tilde{Q} = \tilde{a}\begin{pmatrix} 1 & \tilde{b}' \\ \tilde{c}' & \tilde{d}' \end{pmatrix}$.

(5) Sketches of linear polarizers whose transmission directions are along the horizontal (H), the vertical (V) and the $+45°$ directions are shown in Fig. 3c(c). Write down their matrices $\tilde{L}_\mathrm{H}$, $\tilde{L}_\mathrm{V}$, $\tilde{L}_{+45°}$ respectively.



(a) 全反镜 (b) 1/4 波片(快轴 −45°) (c) 线偏振片

图 3c

Labels: 全反镜 = *total reflector;* 慢轴 = *slow axis;* 快轴 = *fast axis;* +45°线偏振 = +45° *linear polarization;* (a) 全反镜 = *(a) total reflector;* (b) 1/4 波片(快轴 −45°) = *(b) quarter-wave plate (fast axis at* −45°*);* (c) 线偏振片 = *(c) linear polarizer;* 图 3c = *Fig. 3c.*

Part two: quadrant interference.

An interferometer is an instrument that measures a change of phase through the change of light intensity. An ordinary interferometer (such as the Michelson interferometer) measures only the light intensity, from which the magnitude of the phase difference $\Delta\phi$ can be determined, but not its sign; to decide the sign of $\Delta\phi$ one must also observe how the fringes move in and out. This is inconvenient in some applications. Figure 3d shows a sketch of a quadrant interferometer, with which the definite value of $\Delta\phi$ ($-\pi < \Delta\phi \leq \pi$) can be obtained by measuring only the interference intensities at the detectors A and B.

The laser light passes through a linear polarizer (V) whose transmission direction is vertical (perpendicular to the plane of the page), is reflected by total reflector 1 and enters the half-transmitting half-reflecting beam splitter 1, where it is divided into optical path 1 and optical path 2. The light of optical path 1 is reflected by total reflector 5, then passes through the quarter-wave plate whose fast axis is along $-45°$ and enters the half-transmitting half-reflecting beam splitter 2, where it is again split into two beams, which pass through the linear polarizer (V) to detector A and through the linear polarizer (H) to detector B respectively. The light of optical path 2 is reflected by total reflectors 2, 3 and 4, passes through a linear polarizer whose transmission direction is along $+45°$, and is then split by beam splitter 2 into two beams that go to detectors A and B.

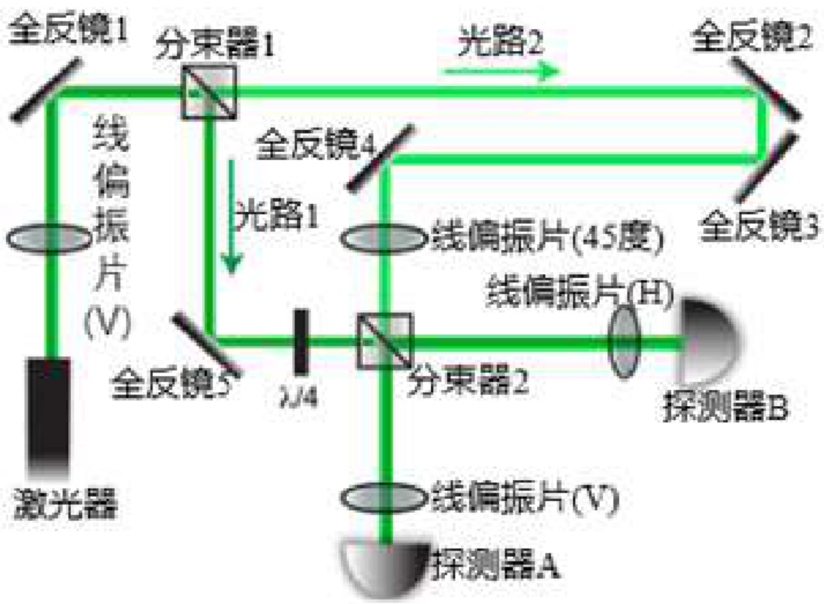


图 3d： 四象限干涉仪示意图

Labels: 全反镜1--5 = *total reflectors 1–5;* 分束器1, 2 = *beam splitters 1, 2;* 线偏振片 = *linear polarizer;* (45度) = *(45°);* 光路1, 光路2 = *optical path 1, optical path 2;* 激光器 = *laser;* 探测器A, 探测器B = *detector A, detector B;* 图 3d：四象限干涉仪示意图 = *Fig. 3d: sketch of the quadrant interferometer.*

(6) The amplitude of the laser light emitted by the laser, just before it reaches beam splitter 1, is E_0. Let l_1, l_2 be the optical paths from beam splitter 1 to beam splitter 2 along optical paths 1 and 2, and let k be the magnitude of the vacuum wave vector of the laser light. Write down the interference terms $I_\mathrm{A}^\mathrm{int}$ and $I_\mathrm{B}^\mathrm{int}$ in the intensity at detector A and in the intensity at detector B (the interference term is the contribution to the intensity caused by interference, i.e. the intensity when both beams are present minus the intensities when each beam is present alone). Then denote by $\Delta\phi$ the total phase appearing in the result for $I_\mathrm{A}^\mathrm{int}$, and express $I_\mathrm{A}^\mathrm{int}$ and $I_\mathrm{B}^\mathrm{int}$ in terms of $\Delta\phi$.[1]

(7) Describe briefly a method of obtaining $\Delta\phi$ from the numerical values of $I_\mathrm{A}^\mathrm{int}$ and $I_\mathrm{B}^\mathrm{int}$.

Neglect all losses of light during propagation.

Problem 4 (70 points)

Consider a simplified model of a manned two-wheeled self-balancing vehicle on rigid, rough, horizontal ground, as shown in Fig. 4a. The model contains two identical rigid circular wheels (of negligible thickness); each wheel has radius r and mass m, uniformly distributed on the rim of the wheel. The centres of the two wheels are joined by a thin, straight, rigid axle of length d and negligible mass, and the plane of each wheel always stays perpendicular to the axle. All parts of the vehicle other than the two wheels and the axle, together with the person standing on the vehicle, are simplified into a uniform rigid thin rod of mass M and length L. The lower end of the rod is connected to the centre C of the axle, and the rod can rotate about C without friction. In addition, there may be a couple (that is, a torque, supplied by the motor on the rod) along the direction of the axle between the rod and the axle. Assume that the wheels always roll without slipping on the ground. Neglect all frictional resistance inside the vehicle system, air resistance and energy losses due to deformation. Take the positive z direction to be vertically upwards, the x axis parallel to the axle and the y axis parallel to the direction of travel of the vehicle. The magnitude of the acceleration due to gravity is g.

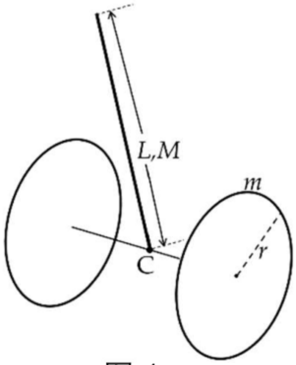


图 4a

Labels: 图 4a = *Fig. 4a.*

(1) In this part the motor delivers no torque, and the contact between the rod and the axle may be regarded as a point contact; assume that the two wheels are not rigidly connected to the

[1] *Translator's note:* The superscript int stands for the Chinese 干涉 ("interference") used in the original.

axle (they can rotate independently of each other), and that the mass of the wheels may be neglected.

Consider the vehicle going round a bend, with the point C moving in uniform circular motion in the horizontal plane with angular velocity Ω. The rod always stays in the vertical plane containing the axle, and its inward inclination angle ϕ stays constant; the rear view is shown in Fig. 4b. Find

(1.1) the radius $R(\phi)$ of the trajectory of the point C;

(1.2) the magnitudes N_{left} and N_{right} of the normal forces exerted by the ground on the left and on the right wheel (the final expressions must not contain R).

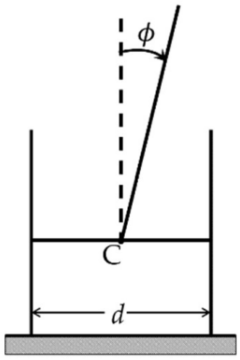


图 4b

Labels: 图 4b = *Fig. 4b.*

In the following parts (2) and (3), the rod moves in the plane perpendicular to the axle; the two wheels are rigidly connected to the axle, and the mass of the wheels may not be neglected.

(2) Consider the vehicle accelerating forward along a straight line; a side view of the vehicle is shown in Fig. 4c. Let the rod be in the position with forward inclination angle θ. In order to keep θ constant, the motor must apply a suitable torque $\tau(\theta)$ to the axle. Find $\tau(\theta)$ and the acceleration $a(\theta)$ of the centre of mass of the vehicle. Let the positive y direction point along the direction of travel of the vehicle, and let the positive sense of the torque be the counter-clockwise sense in Fig. 4c.

图 4c

Labels: 扭矩 = *torque;* 图 4c = *Fig. 4c.*

(3) Consider small oscillations of the vehicle. Suppose the torque $\tau(\theta)$ delivered by the motor obeys the functional relation found in part (2), with all the parameters in that relation still equal to their values in part (2). In this part, however, the mass of the rod is changed to M'. For suitable values of M', the inclination angle θ of the rod (near $\theta = 0$) and the point C can perform small oscillations of the same frequency. Find the condition that M' must satisfy, and the angular frequency ω of these small oscillations.

Problem 5 (50 points)

Consider a spherically symmetric star whose matter is distributed inside a sphere of radius R, with total mass M and centre O. Assume that the stellar matter has only two components: first, neutral or ionized atoms, called the atomic matter; second, radiation, called the photon gas. The gravitational constant is G.

Assume that the total energy of the star comes mainly from the atomic matter, so that radiation may be neglected in parts (1), (2), (3) and (4) below.

(1) First study the hydrostatic equilibrium of the star; the stellar matter may then be regarded as a fluid. Let r be the distance from any point inside the star to its centre O, let the pressure of the fluid at that point be $p(r)$ and its mass density be $\rho(r)$, and let the total mass inside the sphere of radius r be $m(r)$. Write down the differential equation satisfied by $m(r)$ and $p(r)$ in the equilibrium state.

(2) Using the result of part (1), prove that $m(r)$ and $p(r)$ satisfy the inequality

$$\frac{\mathrm{d}}{\mathrm{d}r}\left[p(r) + \frac{Gm^2(r)}{8\pi r^4}\right] < 0,$$

and use this to estimate a lower bound for the pressure at the centre O of the star; express the result in terms of M, R and G.

(3) Calculate the gravitational potential energy of the star itself, and express it as an integral of $p(r)$. Take the zero of potential energy to be the state in which all the stellar matter has been pulled out to infinity in a spherically symmetric way.

(4) Consider the thermodynamics of the stellar matter. Assume that the internal energy density $u(r)$ and the pressure $p(r)$ of the stellar matter satisfy the relation

$$(\gamma - 1)u(r) = p(r),$$

where γ is the adiabatic index. Consider only the case $\gamma > 1$ with γ independent of r. If the star is not to fall apart, what condition must γ satisfy?

The remaining parts take the effects of radiation into account. Assume that the photons inside the star still propagate with the vacuum speed of light c.

(5) Assume the photon gas inside the star is in local thermal equilibrium everywhere; the internal energy of the photon gas comes from the energy of the photons, and its pressure comes from the radiation pressure of the photons. Calculate the adiabatic index γ of such a photon gas.

(6) Atomic matter can absorb photons. When radiation passes through a thin layer of matter of thickness $\mathrm{d}r$ and density ρ, some of the photons are absorbed. The fraction of absorbed photons is $\kappa(r)\rho(r)\,\mathrm{d}r$, and the remaining photons pass through this thin layer of matter; here $\kappa(r)$ is called the opacity. Assume that $\kappa(r)$ does not depend on the frequency of the

light. The atomic matter absorbs photons and therefore experiences a radiation pressure, and this produces a pressure difference in the photon gas. Let the power (luminosity) radiated outwards by the star through the whole spherical surface of radius r centred on O be $L(r)$. Write down the differential equation satisfied by the temperature $T(r)$; the result may contain $L(r)$, $m(r)$, $\kappa(r)$, r and the relevant constants.

(7) Assume that the partial pressure of the atomic matter inside the star decreases as the radius r increases. Estimate an upper bound for the luminosity $L(R)$ at the surface of the star; express the result in terms of M, $\kappa(R)$ and the relevant constants.

Hint: you may use the Planck black-body radiation energy spectrum

$$B(\nu,T)=\frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/(kT)}-1}$$

and the integral formula

$$\int_0^\infty \frac{z^3\,\mathrm{d}z}{e^z-1}=\frac{\pi^4}{15}.$$

Problem 6 (50 points)

According to the uncertainty relation of quantum mechanics $\Delta x\,\Delta p\geq\frac{\hbar}{2}$ (here Δx and Δp are the uncertainties of the displacement x and of the momentum p of the particle, $\hbar=\frac{h}{2\pi}$ is the reduced Planck constant and h is Planck's constant), the displacement x and the momentum p of a harmonic oscillator cannot both be zero, so its ground-state energy cannot be zero. For a similar reason, the energy of the lowest state of the quantized electromagnetic field in vacuum (that is, the state with no photons) cannot be zero either. This leads to an interesting phenomenon: if two uncharged conducting plates are placed parallel to each other in vacuum, a certain distance apart, there is a non-zero electromagnetic interaction force between them.

We now use a simple model to examine the origin of this force. The model considers only the propagation of electromagnetic waves in one-dimensional space, and assumes that the electromagnetic waves have only one polarization mode. Three identical uncharged ideal thin conducting plates A, B, C are placed parallel to one another; the distance between A and B is a and the distance between A and C is L ($L\gg a$), as shown in Fig. 6a. The speed of light in vacuum is c.

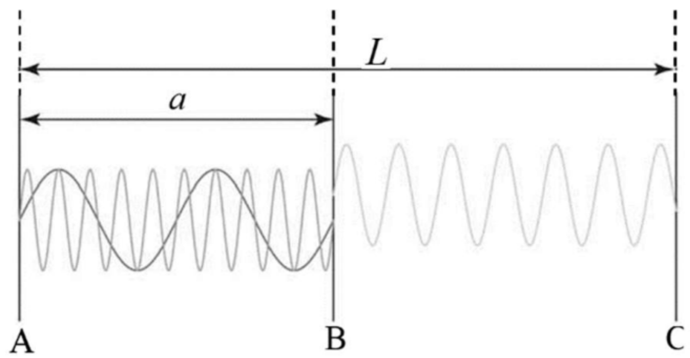


图 6a

Labels: 图 6a = *Fig. 6a.*

(1) The vacuum electromagnetic waves form standing waves between A and B, with the positions of the plate surfaces being nodes. Find the allowed angular frequencies of the electromagnetic

waves between A and B. Similarly, find the allowed angular frequencies of the electromagnetic waves between B and C.

(2) Because of quantum effects, the ground-state energy of a harmonic oscillator of angular frequency ω is $\frac{1}{2}\hbar\omega$. Similarly, the vacuum electromagnetic field energy (zero-point energy) contributed by one mode of angular frequency ω in vacuum is $\frac{1}{2}\hbar\omega$. Write down an expression for the total vacuum electromagnetic field energy between A and C. (Hint: the expression may be divergent, that is, not finite.)

(3) Derive an expression for the electromagnetic interaction force experienced by the conducting plate B.

(4) The magnitude of a real force in nature is always finite. Our result diverges because the assumptions we used are too simplified. For instance, a real conducting plate is not an ideal conductor, and electromagnetic waves of sufficiently high frequency can always pass through the thin conducting plate, so that they are not constrained by the standing-wave condition. Such high-frequency electromagnetic modes are therefore almost unaffected by the positions of the plates, and do not contribute to the electromagnetic force we want to compute. To take this effect into account, we may proceed as follows: we still keep the standing-wave condition found in part (1), but we assume that the effective contribution of each electromagnetic mode to the total electromagnetic field energy is $\frac{1}{2}\hbar\omega\, e^{-\frac{\omega}{\pi\Lambda}}$. Here Λ has the same dimension as ω and satisfies $\frac{\Lambda a}{c} \gg 1$. Calculate the electromagnetic force on the conducting plate B in the limit $L \to \infty$, $\Lambda \to \infty$.

(5) In part (4) a particular exponential function was used to suppress the contribution of the high-frequency electromagnetic waves, and a finite electromagnetic force was obtained; the magnitude of this electromagnetic force, however, does not depend on the particular choice of the function. Therefore consider a more general function $f(x)$, which need only satisfy two conditions:

A. $f(0) = 1$;

B. as $x \to \infty$, $|f(x)|$ tends to zero faster than $\frac{1}{x}$;

the contribution of each electromagnetic mode to the total electromagnetic field energy is $\frac{1}{2}\hbar\omega\, f\left(\frac{\omega}{\pi\Lambda}\right)$. Calculate the electromagnetic force on the conducting plate B in the limit $L \to \infty$, $\Lambda \to \infty$, and show that the result does not depend on the particular form of the function $f(x)$.

(Hint: the Euler–Maclaurin formula is

$$\sum_{n=1}^{N} F(n) - \int_0^N F(n)\,\mathrm{d}n = \frac{F(N) - F(0)}{2} + \frac{F'(N) - F'(0)}{12} + \cdots + B_j\,\frac{F^{(j-1)}(N) - F^{(j-1)}(0)}{j!} + \cdots$$

where $F^{(j)}$ is the j-th derivative of F and B_j are the Bernoulli numbers. If you need this formula, you may keep only the first two terms.)