Разбалловка

T [Условие]

S [Решение]

M [Разбалловка]

ISPhO2026

Русский

Σ 0.0

Сумма баллов

A1 0.20 Determine the pressure p_1 at the bottom of the vessel. Express your answer in terms of m, g, and S.

1 Answer obtained $$p_1 = \frac{mg}{S}.$$	0.20	

A2 0.30 Determine the pressure $p(z)$ on the vessel walls as a function of depth z. Express the answer in terms of ρ, g, and z.

1 Answer obtained $$p = \rho g z.$$	0.30	

B1 0.50 Write down the equilibrium condition for this layer. The equation may include the pressures $P(z)$ and $P(z + \mathrm{d}z)$, $\mathrm{d}z$, the average pressure on the vessel walls $\sigma(z)$, the mass of the layer $\mathrm{d}m$, as well as μ, g, and R.

1 The forces are correctly placed in the figure (automatically graded upon correct writing of the equilibrium condition).	0.20	
2 Answer obtained $$(P(z + \mathrm{d}z) - P(z))\pi R^2 - \mathrm{d}mg + \mu\sigma(z) \cdot 2\pi R\mathrm{d}z = 0.$$	0.30	
3 The equilibrium condition contains errors in defining the surface area on which the pressure force acts, or in the sign of the projection on the chosen axis or a constant factor was lost.	3 × -0.10	0

B2 1.20 Show that the bottom pressure P as a function of the fill height h of the granular column can be written as:

$$P = A + Be^{-h/\lambda}.$$

Determine the parameters A, B, and λ. Express your answers in terms of the bulk density ρ, as well as g, R, K, and μ.

1 The mass of the layer $\mathrm{d}m$ is expressed in terms of $\mathrm{d}z$: $$\mathrm{d}m = \rho\pi R^2\mathrm{d}z.$$	0.20	
2 The following relation was used: $$\sigma(z) = KP(z).$$	0.20	

3 A differential equation following from the equilibrium condition stated in item **B1** has been obtained, for example: $$\frac{\mathrm{d}P}{\mathrm{d}z}+\frac{2K\mu P(z)}{R}-\rho g=0.$$ Points are awarded even if the equilibrium condition in item **B1** is incorrect.	0.30	
4 It is used that $P(0)=0$.	0.20	
5 Answer obtained $$A=\frac{\rho g R}{2\mu K}.$$	0.10	
6 Answer obtained $$B=-\frac{\rho g R}{2\mu K}.$$	0.10	
7 Answer obtained $$\lambda=\frac{R}{2\mu K}.$$	0.10	

B3 1.20 Determine the bulk density of the granules ρ and the coefficient K. Give the answers to two significant figures. For this part, assume the numerical value of the vessel radius is $R=50\ \mathrm{cm}$ and the coefficient of friction is $\mu=0.30$.

1 M1 The pressure dependence on the bottom at small heights is approximated by a linear dependence $$P(h)\approx\rho g h.$$	0.20	
2 M1 The slope of the tangent to the $P(h)$ graph at the point $(0,0)$ has been found: $$k_1\in[6.0,7.0]\ \mathrm{kPa/m}.$$ 3 M1 Dimensional error	0.20 0.15	
4 M1 An expression for the bulk density of granules has been obtained $$\rho=\frac{k_1}{g}.$$	0.10	
5 M1 Answer obtained: $$\rho\in[610,710]\ \mathrm{kg/m^3}.$$ 6 M1 Dimensional error	0.15 0.10	
7 M1 It is suggested to use the value of the steady-state pressure P_∞ at the bottom for large h.	0.20	
8 M1 The value of the steady-state pressure has been determined: $$P_\infty\in[26.5,27.0]\ \mathrm{kPa}.$$ 9 M1 Dimensional error	0.10 0.05	
10 M1 An expression for the coefficient K has been obtained $$K=\frac{\rho g R}{2\mu P_\infty}.$$	0.10	
11 M1 Answer obtained: $$K\in[0.18,0.22].$$	0.15	
12 M2 The idea of using points from the $P(z)$ graph	0.20	

13 M2 It is suggested to determine ρ and K by taking 2 or more points from the $P(z)$ graph.	2 × 0.10	0
14 M2 Two independent equations have been formulated to determine ρ and K.	2 × 0.15	0
15 M2 Answer obtained: $\rho \in [610, 710]\ \mathrm{kg/m^3}.$	0.25	
16 M2 Dimensional error	0.20	
17 M2 Answer obtained: $K \in [0.18, 0.22].$	0.25	

C1 [0.10] In the specified approximation, obtain the dependence of the vessel's cross-sectional radius on height $r(z)$. Express the answer in terms of R, α, and z.

1 Answer obtained: $r(z) = R - \alpha z.$	0.10	
2 A constant factor was lost	0.05	

C2 [0.60] Write down the equilibrium condition for the layer. The equation may include the pressures $P(z)$ and $P(z+\mathrm{d}z)$, $\mathrm{d}z$, the average pressure on the vessel walls $\sigma(z)$, the mass of the layer $\mathrm{d}m$, as well as μ, g, $r(z)$, $r(z+\mathrm{d}z)$, α, and R.

1 The forces are correctly placed in the figure (automatically graded upon correct writing of the equilibrium condition).	0.20	
2 Answer obtained: $P(z+\mathrm{d}z)\cdot \pi r^2(z+\mathrm{d}z) - P(z)\cdot \pi r^2(z) + (\mu+\alpha)\sigma(z)\cdot 2\pi r(z)\mathrm{d}z - \mathrm{d}mg = 0.$	0.40	
3 The equilibrium condition contains errors in defining the surface area on which the pressure force acts, or in the sign of the projection on the chosen axis or a constant factor was lost.	4 × -0.10	0

C3 [0.80] Determine the parameters A, γ, and β. Express the answer in terms of the bulk density of the granules ρ, g, $z_{\max}$, K, μ, and α.

1 A differential equation following from the equilibrium condition stated in item **C2** has been obtained, for example: $\frac{\mathrm{d}(Pr^2)}{\mathrm{d}z} + 2K\mu P(z) r(z) - \rho g r^2(z) = 0.$ Points are awarded even if the equilibrium condition in item **C2** is incorrect.	0.10	
2 It is used that $P(0) = 0$.	0.10	
3 Answer obtained: $\gamma = 1.$	0.10	
4 The general solution, taking $\gamma = 1$ into account, has been substituted into the obtained differential equation, resulting in an equation such as: $\alpha^2 A z_{\max}\left(-1+\beta(1-\xi)^{\beta-1}\right)(1-\xi)^2 + {} +2(K\mu-\alpha)A\alpha z_{\max}(1-\xi)\left(1-\xi-(1-\xi)^{\beta}\right) - \rho g\alpha^2 z_{\max}^2(1-\xi)^2 = 0,$ where $\xi = z/z_{\max}$ or similar.	0.10	
5 From the equality of the coefficients of $(1-\xi)^2$, an equation has been obtained, for example: $-A\alpha^2 + 2(K\mu-\alpha)A\alpha - \rho g\alpha^2 z_{\max} = 0.$	0.10	

6 Answer obtained: $$A = \frac{\rho g \alpha z_{\max}}{2K\mu - 3\alpha}.$$	0.10	
7 From the equality of the coefficients of $(1-\xi)^{\beta+1}$, an equation has been obtained, for example: $$\alpha^2 A\beta - 2(K\mu - \alpha)A\alpha = 0.$$	0.10	
8 Answer obtained: $$\beta = \frac{2(K\mu - \alpha)}{\alpha}.$$	0.10	

C4 0.80 Sketch a qualitative graph of $P(z)$ and determine the ratio of the maximum vertical pressure $P_{\max}$ to $\rho g z_{\max}$ for $\alpha = 0.2K\mu$. Provide the answer to two significant figures.

1 The graph passes through the point $(0, 0)$.	0.10	
2 The graph passes through the point $(z_{\max}, 0)$.	0.20	
3 There is exactly one extremum on the graph. it is a maximum.	0.10	
4 The graph is approximately linear on the initial and final segments.	0.10	
5 The derivative $\dfrac{\partial P(z)}{\partial z}$ for its respective expression has been correctly calculated.	0.10	
6 The root of the equation $\dfrac{\partial P(z)}{\partial z} = 0$ has been correctly found.	0.10	
7 Answer obtained: $$P_{\max} \approx 0.093\rho g z_{\max}.$$	0.10	

D1 0.40 Write down the equilibrium condition for the layer. The equation may include the pressures $P(z)$ and $P(z+\mathrm{d}z)$, $\mathrm{d}z$, the average pressure on the rod and vessel walls $\sigma(z)$, the mass of the layer $\mathrm{d}m$, as well as μ_1, μ_2, g, a, and R.

1 It is stated or used that the friction force acting on the granules from the side of the rod is directed upwards, while the friction force acting on the granules from the side of the vessel is directed downwards.	0.20	
2 Answer obtained: $$(P(z+\mathrm{d}z) - P(z)) \cdot \pi(R^2 - a^2) - \mathrm{d}mg - \mu_1\sigma(z) \cdot 2\pi R\mathrm{d}z + \mu_2\sigma(z) \cdot 2\pi a\mathrm{d}z = 0.$$ Points are awarded even if the direction of the friction forces is incorrect.	0.20	

D2 0.30 Show that the bottom pressure P as a function of the fill height h of the granular column can be written as:

$$P = A + Be^{-h/\lambda}.$$

Determine the parameters A, B, and λ. Express the answers in terms of the bulk density of the granules ρ, as well as g, a, R, μ_1, μ_2, and K.

1 Answer obtained: $$A = \frac{\rho g(R^2 - a^2)}{2(\mu_2 a - \mu_1 R)K}.$$	0.10	
2 Answer obtained: $$B = -\frac{\rho g(R^2 - a^2)}{2(\mu_2 a - \mu_1 R)K}.$$	0.10	

3	Answer obtained: $$\lambda = \frac{R^2 - a^2}{2(\mu_2 a - \mu_1 R)K}.$$	0.10	

D3 0.80 Determine the minimum level H of the granules relative to the bottom at which the cylinder can be lifted as described above. Assume that $H \ll \lambda$. Express the answer in terms of a, R, μ_2, and K.
Hint: You may use the following approximation:

$$e^x \approx 1 + x + \frac{x^2}{2}, \quad x \ll 1$$

1	It is used that $$F \propto \int P(z)\mathrm{d}z.$$	0.10	
2	An expression for the friction force acting on the rod has been written: $$F = 2\pi a K \mu_2 \int_0^H P(z)\mathrm{d}z.$$	0.20	
3	M1 The integral has been correctly evaluated $$F = 2\pi a K \mu_2 A(H - \lambda(1 - e^{-H/\lambda})).$$	0.10	
4	M2 In the approximation $H \ll \lambda$, the following is used: $$P(z) \approx \rho g z$$	0.10	
5	In the approximation $H \ll \lambda$, the following has been obtained: $$F \approx 2\pi a K \mu_2 A \frac{H^2}{2\lambda}.$$	0.10	
6	The equilibrium condition of the system has been stated $$F = \rho g \pi (R^2 - a^2)H.$$	0.20	
7	Answer obtained: $$H = \frac{R^2 - a^2}{K a \mu_2}.$$	0.10	

E1 0.20 Derive an equation relating S_1, S_2, Δl_1, and Δl_2.

1	Answer obtained: $$S_1 \Delta l_1 = S_2 \Delta l_2.$$	0.20	

E2 0.10 Determine the total energy E (the sum of kinetic and potential energies) of the selected fluid volume $S_1 \Delta l_1$. Express the answer in terms of ρ, S_1, Δl_1, v_1, g, and h_1. Take the potential energy at $h = 0$ as the zero reference level.

1	Answer obtained: $$W_1 = \rho S_1 \Delta l_1 \left(\frac{v_1^2}{2} + g h_1\right).$$	0.10	

E3 0.30 Determine the work A done by pressure forces on the fluid bounded by the chosen stream tube during time $\Delta t = \dfrac{\Delta l_1}{v_1}$. Express the answer in terms of p_1, p_2, S_1, and Δl_1.

1	Answer obtained: $$A = (p_1 - p_2)S_1\Delta l_1.$$	0.30	
2	Incorrect sign in the expression for work.	0.20	

E4 0.30 Using the law of conservation of energy, express the pressure p_2 in terms of p_1, ρ, g, h_1, h_2, v_1, and v_2. The resulting equation is known as the **Bernoulli equation**.

1	The law of conservation of energy is stated: $$A = W_2 - W_1.$$	0.20	
2	Answer obtained: $$p_2 = p_1 - \frac{v_2^2 + 2gh_2 - v_1^2 - 2gh_1}{2}.$$	0.10	

E5 0.30 Determine the fluid outflow velocity $v(h)$ as a function of the height above the orifice. Express the answer in terms of g and h.

1	Answer obtained: $$v = \sqrt{2gh}.$$	0.30	

E6 0.60 Determine the time τ required for all the fluid initially above the orifice to drain through it. Express the answer in terms of g, H, R, and D.

1	An expression for the volumetric flow rate of the outflowing liquid has been written, for example: $$\pi R^2 \dot{h} = -\frac{\pi D^2}{4}\sqrt{2gh}.$$	0.20	
2	Incorrect sign	0.10	
3	An expression for the time τ has been obtained in integral form, for example: $$\tau = \frac{2\sqrt{2}R^2}{D^2\sqrt{g}}\int_0^H \frac{\mathrm{d}h}{\sqrt{h}}.$$	0.20	
4	Answer obtained: $$\tau = \frac{4R^2}{D^2}\sqrt{\frac{2H}{g}}.$$	0.20	
5	A numerical coefficient error was made during the calculations, which does not affect the dimensionality.	0.10	

F1 0.20 Due to internal friction between the granules, the maximum angle at which the free surface of the granular material can deviate from the horizontal is θ_r. What is the maximum wall thickness $\omega_{\max}$ for which the flow of granules from the vessel is possible? Express the answer in terms of D and θ_r.

1	Answer obtained: $$\omega_{\max} = D\operatorname{ctg}\theta_r.$$	0.20	

F2 0.80 Determine q and n.

1	It is concluded that the velocity of the particles exiting the orifice does not depend on the height H.	0.20	
2	Answer obtained: $$q = 0.$$	0.10	

3	It is stated or used that $$\frac{\mathrm{d}m}{\mathrm{d}t} \propto D^2 v.$$	0.20	
4	Based on the principles of uniformly accelerated motion, it is concluded that $$v \propto \sqrt{D}.$$	0.20	
5	Answer obtained: $$n = \frac{5}{2}.$$	0.10	