Решение

T Условие

S Решение

Русский

A0 0.20 Using the given value of the radiation length of air X_0, calculate the splitting length d_e. Express your answer in g/cm^2 and round it to one decimal place.
Note: We will call the number of shower generations n the quantity, which denotes how many successive splittings have occurred since the beginning of the shower development.

Ответ:

$$d_e = X_0 \ln 2 = 25.1 \text{ g/cm}^2$$

A1 0.40 Determine the number of shower generations $n_{\max}$ after which the energy of the particles in the shower becomes less than the critical energy.

Since the energy of the shower particles is halved after each splitting length, let us determine after how many splittings the particle energy becomes smaller than the critical energy:

$$\log_2 \frac{E_\gamma}{E_e^{\text{crit}}} = 6.8 \Rightarrow$$

Ответ:

$$n_{\max} = 7$$

A2 0.40 Determine the geometric distance l_e traveled by the shower (in m) in air by the time the maximum number of particles is reached.

Let us calculate the radiation path length traversed by the shower by the time it reaches the 7th splitting:

$$x_e = d_e \cdot n_{\max} = 175.7 \text{g/cm}^2.$$

For an atmosphere of constant density ρ_0, this radiation path length corresponds to the following geometric distance:

$$l_e = \frac{x_e}{\rho_0} = 1.4 \text{ km}$$

A3 0.20 Determine the maximum number of particles $N_{\max}$ in the shower.

At each splitting, the number of shower particles doubles. Therefore,

Ответ:

$$N_{\max} = 2^{n_{\max}} = 128$$

A4 0.80 Determine the total charge Q_e (in C) carried by the electrons in the shower by the time its development ceases.

At each splitting, a photon converts into an electron–positron pair and disappears, while each electron and positron emits one photon. Accordingly, the number of photons after a splitting is equal to the total number of electrons and positrons before the splitting. The number of positrons and electrons after a splitting increases by the number of photons before the splitting. Let us construct a table showing the numbers of each type of particle as a function of the number of generations.

n	0	1	2	3	4	5	6	7
γ	1	0	2	2	6	10	22	42
e^-	0	1	1	3	5	11	21	43
e^+	0	1	1	3	5	11	21	43

We find that when the shower ceases to develop, the number of electrons in it is $n_{e^-} = 43$.
Therefore, the total charge carried by the electrons is

Ответ:

$$Q_e = n_{e^-} \cdot q_e = -6.9 \cdot 10^{-18} \text{ C}$$

B1 0.40 Determine the maximum number of generations of the hadronic component of the shower, $n_{\text{hadr}}^{\max}$.

The number of generations in the hadronic component of the shower is determined analogously to **A1**. However, in a hadronic shower, the particle energy decreases by a factor of 15 at each splitting:

$$\log_{15} \frac{E_p}{E_\pi^{\text{crit}}} = 2.2 \Rightarrow$$

Ответ:

$$n_{\text{hadr}}^{\max} = 3$$

B2 1.00 Determine the maximum number of generations of the entire shower $n_{\text{EM}}^{\max}$, taking into account the electromagnetic component.

In a hadronic interaction, the particle energy decreases by a factor of 15, whereas in an electromagnetic interaction it decreases by a factor of 2. Therefore, the maximum number of generations is obtained when the shower develops electromagnetically from the very beginning, except for the first hadronic interaction, in which the proton decays and the π^0 meson splits into two photons:

$$\log_2 \frac{E_p/(15 \cdot 2)}{E_e^{\text{crit}}} = 11.4 \Rightarrow$$

Ответ:

$$n_{\text{EM}}^{\max} = 12 + 1 = 13$$

B3 0.60 Determine the fraction of the shower energy carried away by muons and neutrinos, β_{miss}. Round your answer to two decimal places.

The energy carried away by muons and neutrinos is the energy remaining in the charged pions $\pi^\pm$ once their energies have fallen below the critical value. At each hadronic interaction, 6 out of 15 particles become neutral pions π^0, and their energy is transferred to the electromagnetic component. The remaining 9 out of 15 particles stay in the hadronic component.

Therefore, after the hadronic shower development ceases (i.e., after
$n_{\text{hadr}}^{\max} = 3$
splittings), the fraction of energy remaining in the hadronic component is

$$\beta_{\text{miss}} = \left(\frac{9}{15}\right)^{n_{\text{hadr}}^{\max}} = \left(\frac{3}{5}\right)^3 = 0.216.$$

All of these charged pions subsequently decay into muons and neutrinos, carrying away this energy. Thus,

Ответ:

Ответ:

$$\beta_{\rm miss} = 0.22$$

C1 [1.30] Determine the altitude $h_{\rm stop}$ at which the shower development ceases. Express your answer in kilometers and round it to one decimal place.

One hadronic interaction reduces the particle energy by a factor of 15, which corresponds to $\log_2 15 = 3.9$ electromagnetic splittings. A reduction of the energy by a factor of 15 corresponds either to a radiation path length $d_h = 57$ g/cm^2, or to a radiation path length $3.9 \cdot d_e = 97.9$ g/cm^2.
Therefore, the electromagnetic shower stops developing later (or farther away), and $h_{\rm stop}$ should be calculated specifically for the electromagnetic component.

Let us calculate the radiation path length traveled by the shower by the time it stops. This consists of one hadronic interaction, followed by the decay of π^0 into two photons and the remaining electromagnetic splittings:

$$\log_2 \frac{E/(15 \cdot 2)}{E_e^{\rm crit}} = 15.2 \Rightarrow x = d_h + 16 \cdot d_e = 458.6 \text{g/cm}^2.$$

Let us now calculate at what geometric altitude the shower stops developing, taking into account the atmospheric density:

$$x = \int_{h_{\rm stop}}^{\infty} \rho_0 e^{-h/H} dh = \rho_0 H e^{-h_{\rm stop}/H}.$$

Ответ:

$$h_{\rm stop} = H \ln \frac{\rho_0 H}{x} = 6.6 \text{ km}$$

C2 [0.50] Determine what fraction of the shower energy, $\beta_{\rm EM}$, ends up in the electromagnetic component. Round your answer to two decimal places.

By analogy with **B3**, let us find the energy carried away by muons and neutrinos:

$$\log_{15} \frac{E}{E_\pi^{\rm crit}} = 3.14 \Rightarrow n_h = 4.$$

Then all the remaining energy stays in the EM component:

Ответ:

$$\beta_{\rm EM} = 1 - \left(\frac{9}{15}\right)^4 = 0.87$$

C3 [0.20] Estimate the shower radius $R_{\rm stop}$ at the altitude $h_{\rm stop}$. Express your answer in meters and round it to the nearest integer.

Let us calculate the air density at the altitude $h_{\rm stop}$:

$$\rho(h_{\rm stop}) = \rho_0 e^{-h_{\rm stop}/H} = 0.57 \text{kg/m}^3.$$

Substituting this value into the radius formula, we obtain:

Ответ:

$$R_{\rm stop} = \frac{X_0}{\rho_0 e^{-h_{\rm stop}/H}} \cdot \frac{E_s}{E_e^{\rm crit}} = 150 \text{ m}$$

C4 [0.20] Estimate the area of the shower S at the Earth's surface. Express your answer in m^2 and keep 2 significant figures.

Scaling the shower radius down to the Earth's surface while accounting for the change in air density, we obtain

$$R = R_{\rm stop} \frac{\rho}{\rho_0} = 67 \text{m}.$$

Ответ:

$$S = \pi R^2 = 14 \cdot 10^3 \text{ m}^2$$

C5 [0.80] Estimate the characteristic signal duration Δt at a detector located on the Earth's surface at the center of the shower. Express your answer in ns and round it to one decimal place.

Since the particles diverge at an angle $\theta = R_{\rm stop}/h_{\rm stop}$ from the altitude $h_{\rm stop}$, the shower front will have the following path-length difference by the time it reaches the Earth:

$$\Delta = h_{\rm stop} \cdot (1 - \cos\theta) \approx h_{\rm stop} \frac{\theta^2}{2} = \frac{R_{\rm stop}^2}{2h_{\rm stop}} = 1.7 \text{m}.$$

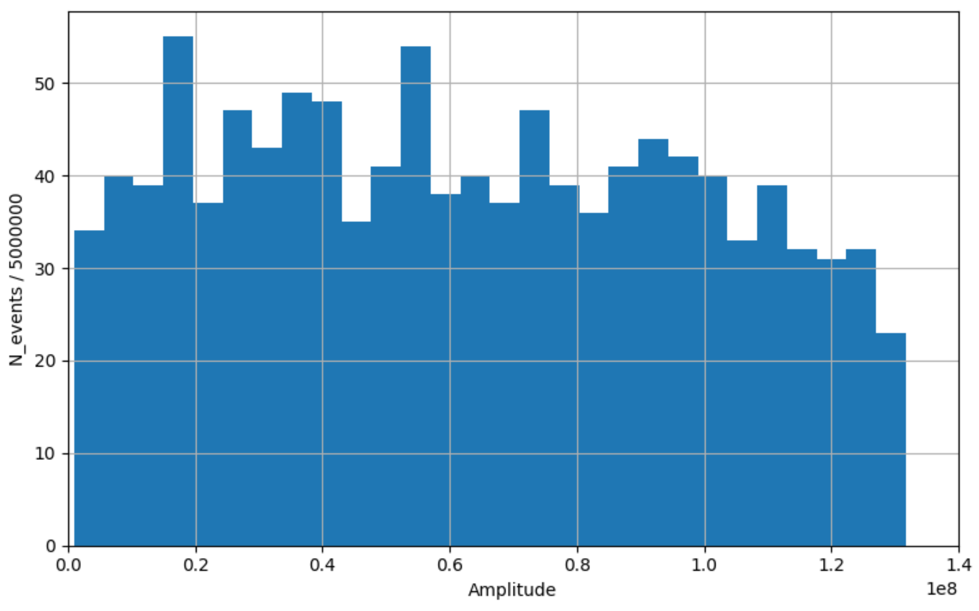
Since the particles travel nearly at the speed of light, we find:

Ответ:

$$\Delta t = \frac{\Delta}{c} = \frac{R_{\rm stop}^2}{2ch_{\rm stop}} = 5.7 \text{ ns}$$

D1 [1.00] Determine the coefficient κ of the detector at the Earth's surface.

To determine κ from the data, let us construct a histogram of the detector signal:



The energy of a particle arriving at the detector cannot exceed the critical energy $E_e^{\rm crit} = 88$ MeV. In the histogram, one can see a sharp cutoff at the signal value $A^{\rm crit} = 1.32 \cdot 10^8$, from which we find:

Ответ:

$$\kappa = \frac{A^{\text{крит}}}{E_e^{\text{крит}}} = 1.5 \cdot 10^9 \text{ GeV}^{-1}$$

D2 [0.50] Determine the energy of the initial photon E_0.

Let us sum all the signal recorded by the detector:

$$A_0 = 70240347424,$$

from which, taking into account the coefficient κ, we obtain:

Ответ:

$$E_0 = \frac{A_0}{\kappa} = 46.8 \text{ GeV}$$

D3 [1.50] Calculate the atmospheric density at the Earth's surface ρ_{atm}.

Let us calculate the mean shower radius from the data using the formula from **C3**:

$$R = \sum_i \frac{\sqrt{x_i^2 + y_i^2}}{N} = 68.3\text{m}.$$

After the shower development ceased, its radius was, according to formula (8),

$$R_{\text{stop}} = \frac{X_0}{\rho(h_{\text{stop}})} \cdot \frac{E_s}{E_e^{\text{crit}}},$$

and then it decreased inversely proportional to the density according to formula (9):

$$R = R_{\text{stop}} \frac{\rho(h_{\text{stop}})}{\rho_{\text{atm}}} = \frac{X_0}{\rho_{\text{atm}}} \cdot \frac{E_s}{E_e^{\text{crit}}}.$$

Ответ:

$$\rho_{\text{atm}} = \frac{X_0}{R} \cdot \frac{E_s}{E_e^{\text{крит}}} = 1.26 \text{ kg/m}^3$$