Решение

T Условие

S Решение

Русский

A1 0.50 Total electric charge near the junction must be equal to zero. Using this fact, write an equation connecting widths of depletion layers d_n, d_p and impurities concentrations N_a, N_d.

Let A denote the contact area. Since the volume charge density is $\rho = \pm q_e N_{a,d}$ ($+$ for holes, $-$ for electrons), the vanishing of the total hole and electron charge implies $AN_a d_p = AN_d d_n$, which yields

Ответ:

$$N_a d_p = N_d d_n.$$

A2 0.50 Show that the following equation holds for electric field

$$\frac{dE_x}{dx} = \frac{\rho}{\varepsilon_0 \varepsilon},$$

where ρ – volume charge density in the corresponding point.

According to the Gauss law for electric fields in the differential form,

$$\operatorname{div}\vec{E} = \frac{\rho}{\varepsilon_0 \varepsilon}.$$

Since $\vec{E} = \vec{E}(x)$, we have

$$\operatorname{div}\vec{E} = \frac{dE_x}{dx},$$

which proves the required statement.

A3 1.00 Find the electric field $E_x(x)$ as a function of x. Assume that far from junction the field is equal to 0. Express the answer in terms of N_a, N_d, d_n, d_p, ε, ε_0 and q_e.

Using the result of part A2, as well as the relationship between concentration and volume charge density $\rho = \pm q_e N_{a,d}$ ($+$ for holes, $-$ for electrons), we get

$$\frac{dE_x}{dx} = \begin{cases} -\frac{q_e N_a}{\varepsilon_0 \varepsilon}, & -d_p \le x \le 0, \\ \frac{q_e N_d}{\varepsilon_0 \varepsilon}, & 0 < x \le d_n, \\ 0, & x < -d_p \text{ or } x > d_n. \end{cases}$$

It remains to take the indefinite integral and find the constant of integration. To obtain the answer faster, we will directly evaluate the definite integral. Since $E_x(\pm\infty) = 0$, we have

$$E_x(x) = \int_{-\infty}^{x} E'(x)\,dx = -\int_{x}^{+\infty} E'(x)\,dx.$$

We use these relations to get the final answer.

Ответ:

$$E_x(x) = \begin{cases} -\frac{q_e N_a (x+d_p)}{\varepsilon_0 \varepsilon}, & -d_p \le x \le 0, \\ \frac{q_e N_d (x-d_n)}{\varepsilon_0 \varepsilon}, & 0 < x \le d_n, \\ 0, & x < -d_p \text{ or } x > d_n. \end{cases}$$

A4 1.00 Find the electrostatic potential $\varphi(x)$ as a function of x. Assume that $\varphi(0) = 0$. Express the answer in terms of N_a, N_d, d_n, d_p, ε, ε_0 and q_e. Determine the potential difference at large distance to the junction $\Delta\varphi = \varphi(x = +\infty) - \varphi(x = -\infty)$.

In the one-dimensional case, $\varphi'(x) = -E_x(x)$. Using the condition $\varphi(0) = 0$, we can express the electrostatic potential as:

$$\varphi(x) = -\int_0^x E_x(x)\mathrm{d}x.$$

Evaluating the integral explicitly, we obtain:

Ответ:

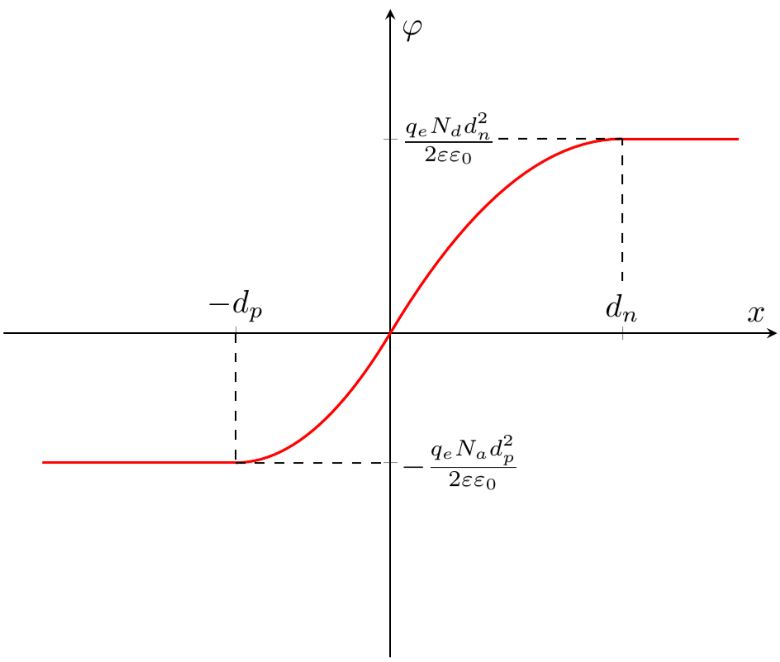
$$\varphi(x) = \begin{cases} -\frac{q_e N_a d_p^2}{2\varepsilon_0\varepsilon}, & x < -d_p, \\ \dfrac{q_e N_a(x^2 + 2d_p x)}{2\varepsilon_0\varepsilon}, & -d_p \le x < 0, \\ \dfrac{q_e N_d(2d_n x - x^2)}{2\varepsilon_0\varepsilon}, & 0 \le x < d_n, \\ \dfrac{q_e N_d d_n^2}{2\varepsilon_0\varepsilon}, & x \ge d_n. \end{cases}$$

Ответ:

$$\Delta\varphi = \frac{q_e(N_a d_p^2 + N_d d_n^2)}{2\varepsilon\varepsilon_0}.$$

A5 0.50 Draw a qualitative plot of the potential $\varphi(x)$ as function of x.

Ответ:



A6 1.00 Actually, the potential difference is determined by the material properties and the temperature. Knowing the potential difference, one can calculate depletion layers widths. Express the widths d_n, d_p in terms of $\Delta\varphi$, N_a, N_d, ε, ε_0 and q_e. Calculate their numerical values for $\Delta\varphi = 0.8$ V.

From part A4 we have

$$\Delta\varphi = \frac{q_e(N_a d_p^2 + N_d d_n^2)}{2\varepsilon\varepsilon_0}.$$

Using the equation $N_a d_p = N_d d_n$ obtained in part A1 we get

$$\Delta\varphi = \frac{q_e\left(N_a d_p^2 + \frac{N_a^2}{N_d} d_p^2\right)}{2\varepsilon\varepsilon_0} = \frac{q_e N_a (N_a + N_d) d_p^2}{2\varepsilon\varepsilon_0 N_d}.$$

Similarly we have

$$\Delta\varphi = \frac{q_e\left(\frac{N_d^2}{N_a} d_n^2 + N_d d_n^2\right)}{2\varepsilon\varepsilon_0} = \frac{q_e N_d (N_a + N_d) d_n^2}{2\varepsilon\varepsilon_0 N_a}.$$

what implies the answer:

Ответ:

$$d_p = \sqrt{\frac{2\varepsilon\varepsilon_0 \Delta\varphi N_d}{q_e N_a (N_a + N_d)}} \approx 58.7\ \mathrm{nm},$$

$$d_n = \sqrt{\frac{2\varepsilon\varepsilon_0 \Delta\varphi N_a}{q_e N_d (N_a + N_d)}} \approx 117.5\ \mathrm{nm}.$$

A7 0.50 Determine the maximum value of the electric field in the junction $E_{\max}$. Express the answer in terms of $\Delta\varphi$, N_a, N_d, ε, ε_0 and q_e. Calculate its numerical value for $\Delta\varphi = 0.8\ \mathrm{V}$.

It follows from the $E_x(x)$ dependence obtained in part A3 that the maximal absolute value of the electric field is achieved at $x = 0$ and equals

$$E_{\max} = \frac{q_e N_a d_p}{\varepsilon\varepsilon_0}.$$

Substituting the expression for d_p obtained in part A6 we get

Ответ:

$$E_{\max} = \sqrt{\frac{2 q_e \Delta\varphi N_a N_d}{\varepsilon\varepsilon_0 (N_a + N_d)}} \approx 9.08 \cdot 10^6\ \mathrm{V/m}.$$

B1 0.50 Let n_{10} denote the concentration of electrons in p-type semiconductor without external voltage. Similarly let p_{10} denote the concentration of holes in n-type semiconductor without external voltage. Determine electron concentration n_1 in p-type semiconductor and hole concentration p_1 in n-type semiconductor after the voltage V is applied. Consider region outside of depletion layers, where electric field is zero. Express the answers in terms of p_{10}, n_{10}, V, T, k and q_e.

It follows from the formula for the distribution of $n(x)$ that

$$\frac{n(x=+\infty)}{n(x=-\infty)} = \exp\left(-\frac{U_n(x=+\infty) - U_n(x=-\infty)}{kT}\right)$$

Before the voltage is applied, we have $n(x=+\infty) = N_d$, $n(x=-\infty) = n_{10}$. After the voltage is turned on, the difference of potential energies $U_n(x=+\infty) - U_n(x=-\infty)$ for a single electron changes by $(-q_e)\cdot(-V) = q_e V$. According to the problem text, the electron concentration in the n-type semiconductor equals N_d for any voltage. So after applying the voltage we still have $n(x=+\infty) = N_d$, but $n(x=-\infty) = n_1$. Thus we get

$$\frac{N_d}{n_1} = \exp\left(-\frac{q_e V}{kT}\right)\cdot\frac{N_d}{n_{10}},$$

which implies

$$n_1 = n_{10}\cdot\exp\left(\frac{q_e V}{kT}\right).$$

The expression for p_1 can be obtained similarly. It follows from the formula for the distribution of $p(x)$ that

$$\frac{p(x=+\infty)}{p(x=-\infty)} = \exp\left(-\frac{U_p(x=+\infty) - U_p(x=-\infty)}{kT}\right).$$

Before the voltage is applied, we have $p(x=+\infty) = p_{10}$, $p(x=-\infty) = N_a$. After the voltage is turned on, the difference of potential energies $U_p(x=+\infty) - U_p(x=-\infty)$ for a single hole changes by $q_e\cdot(-V) = -q_e V$. According to the problem text, the concentration of holes in the p-type semiconductor equals N_a, so that after applying the voltage we still have $p(x=-\infty) = N_p$, but $p(x=+\infty) = p_1$. Thus we get

$$\frac{p_1}{N_a} = \exp\left(\frac{q_e V}{kT}\right) \cdot \frac{p_{10}}{N_a},$$

which implies

$$p_1 = p_{10} \cdot \exp\left(\frac{q_e V}{kT}\right).$$

Ответ:

$$n_1 = n_{10} \cdot \exp\left(\frac{q_e V}{kT}\right), \quad p_1 = p_{10} \cdot \exp\left(\frac{q_e V}{kT}\right).$$

B2 1.00 Consider the volume between the planes with coordinates x and $x + \Delta x$ (Fig. 3). Write down the condition that the holes concentration in this volume does not change. Take into account the holes flux through the two planes and holes recombination. In the limit $\Delta x \to 0$ obtain a differential equation for the concentration deviation from equilibrium value $\Delta p = p_1 - p_{10}$ as a function of x. The answer may also include D_p and τ_p.

Let A denote the cross section area of the n-type semiconductor. Consider a small time interval Δt and calculate the change number of holes change within the small volume under consideration over this time interval. The change due to recombination is $\left(\frac{dp_1}{dt}\right)_{\text{rec}} A\Delta x\Delta t$. The changes due to diffusion through the two planes are $J_p(x)A\Delta t$ (at coordinate x) and $-J_p(x+\Delta x)A\Delta t$ (at coordinate $x + \Delta x$). The total number of holes, and therefore concentration, are constant, so the total change must be zero:

$$\left(\frac{dp_1}{dt}\right)_{\text{rec}} A\Delta x\Delta t + J_p(x)A\Delta t - J_p(x+\Delta x)A\Delta t = 0.$$

Thus we get

$$\left(\frac{dp_1}{dt}\right)_{\text{rec}} = \frac{J_p(x+\Delta x) - J_p(x)}{\Delta x} = \frac{dJ_p(x)}{dx} = -D_p \frac{d^2 p_1}{dx^2}.$$

Since Δp and p_1 differ by a constant,

$$\frac{d^2 p_1}{dx^2} = \frac{d^2 \Delta p}{dx^2}.$$

Taking into account the equation

$$\left(\frac{dp_1}{dt}\right)_{\text{rec}} = -\frac{\Delta p}{\tau_p}$$

we obtain:

Ответ:

$$\frac{d^2 \Delta p}{dx^2} = \frac{\Delta p}{\tau_p D_p}$$

B3 0.60 Find the distance L_p, over which the deviation of holes concentration from equilibrium decreases by e times. Express the answer in terms of D_p, τ_p. Compute numerical value for $D_p = 1.0 \cdot 10^{-3}\ \text{m}^2/\text{s}$, $\tau_p = 5.0 \cdot 10^{-7}\ \text{s}$.

The general solution of the equation from part B2 has the following form:

$$\Delta p = a \exp\left(-\frac{x}{\sqrt{\tau_p D_p}}\right) + b \exp\left(\frac{x}{\sqrt{\tau_p D_p}}\right)$$

for some coefficients a and b. As $x \to +\infty$ the hole concentration must approach the equilibrium value p_{10}, so that $\Delta p \to 0$, and thus $b = 0$. Therefore,

$$\Delta p \sim \exp\left(-\frac{x}{\sqrt{\tau_p D_p}}\right),$$

and hence Δp changes by a factor of e when x changes by

Ответ:

$$L_p = \sqrt{\tau_p D_p} \approx 2.24 \cdot 10^{-5}\ \text{m}.$$

B4 0.80 Determine the hole concentration $p_1(x)$ in region $x > d_n$ as a function of coordinate (in n-type semiconductor outside of the depletion layer). Express the answer in terms of p_{10}, V, T, L_p, d_n, k and q_e.

From the expression for Δp obtained in part B3, we find for $x > d_n$:

$$p_1(x) = p_{10} + a\exp\left(-\frac{x}{L_p}\right),$$

where a is some coefficient. Within the depletion layer, the concentration $p(x)$ obeys the Boltzmann distribution. Therefore,

$$\frac{p(d_n)}{p(-d_p)} = \exp\left(-\frac{U_p(d_n) - U_p(-d_p)}{kT}\right).$$

According to the problem text, the electric field outside the depletion layer can be assumed to be zero, so the voltage change falls entirely within the region $-d_p \le x \le d_n$. Therefore, when the voltage is applied, the potential energy difference $U_p(d_n) - U_p(-d_p)$ for a single hole changes by $-q_eV$, so that similarly to part B1 we can write

$$\frac{p_1(d_n)}{N_a} = \exp\left(\frac{q_eV}{kT}\right)\frac{p_{10}}{N_a},$$

which implies

$$p_1(d_n) = \exp\left(\frac{q_eV}{kT}\right)p_{10} = p_{10} + a\exp\left(-\frac{d_n}{L_p}\right),$$

so that

$$a = p_{10}\left(\exp\left(\frac{q_eV}{kT}\right) - 1\right)\exp\left(\frac{d_n}{L_p}\right).$$

Ответ:

$$p_1(x) = p_{10} + p_{10}\left(\exp\left(\frac{q_eV}{kT}\right) - 1\right)\exp\left(-\frac{x - d_n}{L_p}\right)$$

B5 0.80 Determine the density of hole diffusion current j_p at the boundary of depletion layer $x = d_n$. Express the answer in terms of p_{10}, V, L_p, D_p, T, k and q_e.

To calculate the current of holes, let us consider the n-type region near the boundary of the depletion layer, where the flux of holes is determined by diffusion:

$$\begin{aligned} J_p = -D_p\frac{dp_1}{dx} &= \frac{D_p p_{10}}{L_p}\left(\exp\left(\frac{q_eV}{kT}\right) - 1\right)\exp\left(-\frac{x - d_n}{L_p}\right)\Bigg|_{x=d_n} = \\ &= \frac{D_p p_{10}}{L_p}\left(\exp\left(\frac{q_eV}{kT}\right) - 1\right). \end{aligned}$$

The current density is $j_p = q_eJ_p$, which yields

Ответ:

$$j_p = \frac{q_eD_p p_{10}}{L_p}\left(\exp\left(\frac{q_eV}{kT}\right) - 1\right).$$

B6 0.80 The junction cross section area is A. Derive the expression for the total current I through the junction as a function of applied voltage V, semiconductor parameters N_a, N_d, τ_p, τ_n, D_p, D_n, n_i and constants k, q_e.

Similarly to part B5, the expression for the electron current has the form

$$j_n = \frac{q_eD_n n_{10}}{L_n}\left(\exp\left(\frac{q_eV}{kT}\right) - 1\right).$$

Taking into account that

$$p_{10} = \frac{n_i^2}{N_d}, \quad n_{10} = \frac{n_i^2}{N_a},$$

и we find the required current as the sum of the hole and electron currents:

$$I = q_e A n_i^2 \left(\exp\left(\frac{q_e V}{kT} \right) - 1 \right) \left(\frac{D_p}{L_p N_d} + \frac{D_n}{L_n N_a} \right).$$

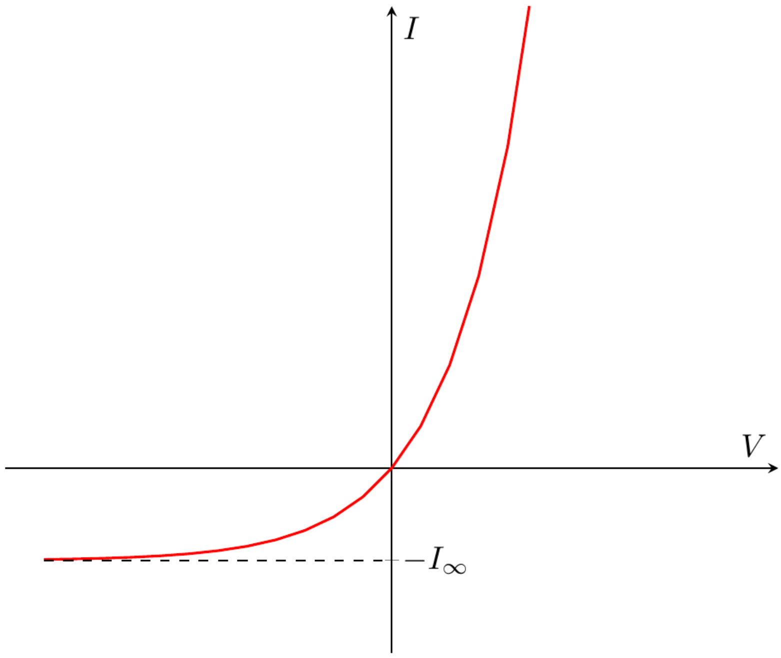
Using the answer in part B3, we obtain:

Ответ:

$$I = q_e A n_i^2 \left(\exp\left(\frac{q_e V}{kT} \right) - 1 \right) \left(\frac{1}{N_d} \sqrt{\frac{D_p}{\tau_p}} + \frac{1}{N_a} \sqrt{\frac{D_n}{\tau_n}} \right).$$

B7 0.30 Draw a qualitative plot of the relation $I(V)$, obtained in previous task.

Ответ:



B8 0.20 Calculate the numerical values of the saturation current I_∞ at large negative voltage, and the value of current I_1 for $V = 0.7$ V.

The saturation current is

$$I_\infty = q_e A n_i^2 \left(\frac{1}{N_d} \sqrt{\frac{D_p}{\tau_p}} + \frac{1}{N_a} \sqrt{\frac{D_n}{\tau_n}} \right) = 5.77 \cdot 10^{-16} \text{ A},$$

and the current at $V = 0.7$ V equals to $I_1 = 3.23 \cdot 10^{-4}$ A.

Ответ:

$$I_\infty = 5.77 \cdot 10^{-16} \text{ A}, \quad I_1 = 3.23 \cdot 10^{-4} \text{ A}.$$