SJPO 2026 Solutions (Unofficial)

FIZIKA
www.projectfizika.com

May 2026

The questions to SJPO 2026 can be found here, hosted on phoXiv.

Note: These solutions are **unofficial**, and were originally produced by FIZIKA. If you spot any errors or any misinterpreted questions, please contact us. Thank you!

Solutions

Question 1

(a) $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ is not a unit vector as its magnitude is $\sqrt{1^2 + 1^2} = \sqrt{2}$.

(b) $\hat{\mathbf{i}} \cdot \hat{\mathbf{i}}$ is not even a vector; it is a scalar and has value 1.

(c) $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \mathbf{0}$ is not a unit vector.

(d) $\left(\hat{\mathbf{i}} \times \hat{\mathbf{j}}\right) \times \hat{\mathbf{k}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = \mathbf{0}$ is not a unit vector.

(e) $\left(\hat{\mathbf{i}} \times \hat{\mathbf{j}}\right) \times \hat{\mathbf{i}} = \hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}$ is a unit vector!

Therefore, the answer is $\boxed{\text{(e)}}$.

Question 2

The mirror always lies at the midpoint between the object and its image. As such, the image must be travelling at speed $\frac{u}{2}$ away the man, so the answer is $\boxed{\text{(a)}}$.

Question 3

Label the runners with their respective sides. While runner a is travelling downwards and runner b is travelling rightwards, their distance is always increasing. While runners a and b are travelling rightwards, their distance does not change as they are moving at the same speeds. Finally, while runner a is travelling rightwards and runner b is travelling downwards, their distance is always decreasing.

Therefore, the maximum distance occurs when they are both travelling rightwards, where runner a is a horizontal distance and vertical distance a from runner b. This maximum distance is then $\sqrt{a^2 + a^2} = a\sqrt{2}$, so the answer is $\boxed{\text{(c)}}$.

Question 4

The ray from the Sun that grazes the top of Shuhao must end up at the end of his shadow, which forms a right triangle of height h and length l. The angle of the Sun from the horizon is then simply $\tan^{-1}\frac{h}{l} \approx 30°$, i.e. the answer is $\boxed{\text{(a)}}$.

Question 5

The bubble displaces fluid of volume V to to rise, so the change in potential energy of the system is the sum of those contributed by the bubble and the displaced fluid. The bubble rises by h while the displaced fluid falls by h, so the change in potential energy is

$$\Delta U = \rho_b V gh - \rho_f V gh = (\rho_b - \rho_f)V gh$$

i.e. the answer is $\boxed{\text{(c)}}$.

Question 6

Let the block have mass m and the ramp have angle θ from the table. Balancing forces along the perpendicular and parallel directions,

$$N = mg\cos\theta$$
$$ma = mg\sin\theta - f$$

The smallest possible value of a is achieved when the block just slips, which occurs when

$$mg\sin\theta = f_s = \mu_s N = \mu_s mg\cos\theta \implies \tan\theta = \mu_s$$

Then, the block will have acceleration

$$a = g\sin\theta - \frac{f_k}{m} = g(\sin\theta - \mu_k\cos\theta) = \frac{\mu_s - \mu_k}{\sqrt{1+\mu_s^2}}g$$

so the answer is $\boxed{\text{(e)}}$.

Question 7

The maximum compression x of the spring occurs when both masses are moving at the same velocity v, since this is when they will have zero relative velocity. By conservation of momentum, this velocity must be

$$mu = 2mv \implies v = \frac{u}{2}$$

Therefore, by conservation of energy,

$$\frac{1}{2}mu^2 = 2\left(\frac{1}{2}mv^2\right) + \frac{1}{2}kx^2 = \frac{1}{4}mu^2 + \frac{1}{2}kx^2 \implies x = \sqrt{\frac{m}{2k}}u = \frac{u\sqrt{2}}{2}\sqrt{\frac{m}{k}}$$

so the answer is $\boxed{\text{(b)}}$.

Question 8

At steady state, the droplet will reach the same speed v before every collision, which is its terminal speed since it is the largest speed it achieves. Then the speed after every collision is $\frac{v}{2}$, so analysing the free fall motion,

$$v^2 - \left(\frac{v}{2}\right)^2 = 2gh \implies v = \sqrt{\frac{8}{3}gh}$$

i.e. the answer is $\boxed{\text{(d)}}$.

Question 9

The left string becomes slack when the mass m becomes far enough such that the centre of mass of the rod-block system lies below the right string. This is because the net gravitational force will exert no torque on the system about the point where the right string is connected to the rod (as the line of force passes through the pivot), so in order for the system to remain at rest, the left string must exert no torque. This means it exerts no force at all, since it is located at a non-zero distance from the pivot.

Therefore, taking the centre of the rod as the origin, we can equate the position of the centre of mass to that of the pivot, so

$$\frac{mx}{M+m} = \frac{l}{2} \implies x = \frac{l}{2}\frac{M+m}{m}$$

i.e. the answer is $\boxed{\text{(a)}}$.

Question 10

I. As we increase b towards its critical value, the period has to increase to infinity, so this statement is false.

II. As we increase m, the rate of decay of oscillations has to decrease, so this statement is false.

III. This statement is true; it happens when $b = 2\sqrt{mk}$.

Therefore, only statement III is true, so the answer is $\boxed{\text{(b)}}$.

Question 11

Since the box has width $\left(2+\sqrt{2}\right)r$, the horizontal distance between the centres of the spheres are $\left(2+\sqrt{2}\right)r - 2r = \sqrt{2}r$. Then the angle the normal force makes with the horizontal is $\theta = \cos^{-1}\left(\frac{\sqrt{2}r}{2r}\right) = 45°$. The vertical component of this normal force must balance gravity on the right sphere (since the normal force by the wall is only exerted horizontally), so

$$N\sin\theta = mg \implies N = mg\sqrt{2}$$

i.e. the answer is $\boxed{\text{(d)}}$.

Question 12

Let the angle that the right side of the string makes with the horizontal be θ. The equations of motion in the horizontal direction for both blocks is

$$\begin{aligned} ma_1 &= T \\ ma_2 &= T\cos\theta \end{aligned}$$

where T is the tension in the string. Since $\cos\theta \leq 1$, we always have $a_1 \geq a_2$, so block 1 takes less time than block 2 to travel the same horizontal distance L. Therefore, block 1 reaches the pulley before block 2 strikes the table, so the answer is $\boxed{\text{(b)}}$.

Question 13

In order to bounce back and forth indefinitely, the ball's velocity must be reversed upon collision to reverse its path. This occurs when the velocity upon collision is perpendicular to the cone's surface, so we just need to find the time t taken for the velocity to point an angle of θ downwards from the horizontal. This occurs when the vertical speed is $u\tan\theta$, so

$$u\tan\theta = gt \implies t = \frac{u}{g}\tan\theta$$

Finally, the period is 4 times this value, since the ball also needs to return to its starting position, travel to the other side and return again. Therefore,

$$T = 4t = \frac{4u}{g}\tan\theta$$

so the answer is $\boxed{\text{(e)}}$.

Question 14

When the string hits the peg, the pendulum does not change in speed v but its length decreases from L to $\frac{L}{4}$. Balancing the net upwards force with the centripetal force before hitting the peg,

$$\frac{mv^2}{L} = T - mg = 2mg$$

Then, balancing forces again after hitting the peg,

$$\frac{mv^2}{\frac{L}{4}} = T' - mg \implies T' = mg + 4(2mg) = 9mg$$

so the answer is $\boxed{\text{(e)}}$.

Question 15

Since the pendulums oscillate in phase and they have the same length, the spring never changes in length, so it exerts no force. Therefore, the period of their oscillations is just that of a standard pendulum of length l, i.e. $2\pi\sqrt{\frac{l}{g}}$, so the answer is $\boxed{\text{(a)}}$.

Question 16

Let the initial velocity of the moving ball be $\mathbf{u}$ and the final velocities of the two balls of mass m be $\mathbf{v}_1, \mathbf{v}_2$. Conservation of momentum and conservation of energy gives

$$m\mathbf{u} = m\mathbf{v}_1 + m\mathbf{v}_2 \implies \mathbf{u} = \mathbf{v}_1 + \mathbf{v}_2$$
$$\frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \implies \mathbf{u}\cdot\mathbf{u} = \mathbf{v}_1\cdot\mathbf{v}_1 + \mathbf{v}_2\cdot\mathbf{v}_2$$

where we used the fact that for a vector $\mathbf{x}$ with magnitude x, $\mathbf{x}\cdot\mathbf{x} = x^2$. Now, taking the dot product of the first equation with itself and subtracting the second equation gives

$$0 = (\mathbf{v}_1 + \mathbf{v}_2)\cdot(\mathbf{v}_1 + \mathbf{v}_2) - (\mathbf{v}_1\cdot\mathbf{v}_1 + \mathbf{v}_2\cdot\mathbf{v}_2) = 2\mathbf{v}_1\cdot\mathbf{v}_2 \implies \mathbf{v}_1\cdot\mathbf{v}_2 = 0$$

This implies that the two velocities are perpendicular, so $\theta_1 + \theta_2 = 90°$. Therefore, $\theta_2 = 90° - \theta_1 = 15°$, so the answer is $\boxed{\text{(a)}}$.

Question 17

Suppose friction f points to the right. Considering the net force gives

$$ma = T + f$$

while considering the net torque about the centre of the cylinder gives

$$I\alpha = \frac{1}{2}mr^2\alpha = Tr - fr \implies \frac{1}{2}mr\alpha = T - f$$

Since the cylilnder rolls without slipping, $a = r\alpha$. Substituting this and the first equation into the second equation,

$$\frac{1}{2}ma = \frac{1}{2}(T + f) = T - f \implies f = \frac{1}{3}T$$

so the answer is $\boxed{\text{(c)}}$.

Question 18

The work done by tension is the sum of the translational work done on the centre of mass and the rotational work done about the centre of mass, which is

$$W = Fx + \tau\theta = Tx + Tr\left(\frac{x}{r}\right) = 2Tx$$

where we used the fact that the cylinder rolls without slipping, so $x = r\theta$. Therefore, the answer is $\boxed{\text{(e)}}$.

Question 19

Angular velocity always points in the direction of the axis of rotation. This is in the direction of **II**, which can be verified using the right hand rule.

On the other hand, angular momentum is defined by $\mathbf{L} = \mathbf{r} \times \mathbf{p}$. In the given diagram, for every point on the rod, $\mathbf{r}$ points downwards along the rod while $\mathbf{p}$ points into the page, so $\mathbf{L}$ must point perpendicular to the rod in the page, in the direction of **III**.

Therefore, $\boldsymbol{\omega}$ is in the direction of **II** and $\mathbf{L}$ is in the direction of **III**, so the answer is $\boxed{\text{(d)}}$.

Question 20

The only force acting on the mass is static friction, and it must provide both the tangential and centripetal forces to keep the mass accelerating in the circle. These forces are

$$F_\mathrm{t} = mR\alpha$$
$$F_\mathrm{c} = mR\omega^2 = mR\alpha^2t^2$$

Both of these forces are perpendicular and must sum to the friction force, so the magnitude of the friction force is

$$f = \sqrt{F_\mathrm{t}^2 + F_\mathrm{c}^2} = mR\sqrt{\alpha^2 + \alpha^4t^4}$$

Finally, when it just slips, since the normal force is just the weight of the mass,

$$f = \mu_sN = \mu_smg = mR\sqrt{\alpha^2 + \alpha^4t^4} \implies t = \left(\frac{\mu_s^2g^2}{R^2\alpha^4} - \frac{1}{\alpha^2}\right)^{\frac{1}{4}}$$

so the answer is $\boxed{\text{(e)}}$.

Question 21

The minimum pressure in the horizontal section of the siphon is zero (since it is not possible to achieve negative pressure). Since the tank is very large, by continuity, the speed of the drop in water is negligible. Using Bernoulli's principle at the water surface in the tank and the exit point of the siphon, the velocity at the exit is

$$P_0 + \rho g l = P_0 + \frac{1}{2}\rho v^2 \implies \frac{1}{2}\rho v^2 = \rho g l$$

At the maximum height, the pressure in the horizontal section of the siphon is zero. Since the siphon's diameter is uniform, by continuity, the velocity in the horizontal section is the same as that at the exit point. Using Bernoulli's principle again at the water surface in the tank and the horizontal section,

$$P_0 = \frac{1}{2}\rho v^2 + \rho g h = \rho g(l + h) \implies h = \frac{P_0}{\rho g} - l$$

so the answer is $\boxed{\text{(c)}}$.

Question 22

A useful trick we can apply here is we can consider the dust particle to have some extremely small tangential velocity and assume it does not collide with Earth. Then the trajectory of the particle will be an extremely elongated ellipse with a focus at the Earth's position. Since $r \ll R$, we essentially need the time taken for the particle to traverse half of its orbit (from its maximum to minimum distance), which would be half of its period!

Kepler's third law of planetary motion states that the square of a planet's orbital period is proportional to the cube of the length of the semi-major axis of the orbit. This actually implies that orbits with the same semi-major axis will have the same orbital period (for the same gravitational force), so the period of the elliptical trajectory taken by the particle, which has semi-major axis $\frac{r}{2}$, is the same as the period of a circular orbit of radius $\frac{r}{2}$ around the Earth! This period is simple to calculate and arises from equating the centripetal force to the gravitational force,

$$\frac{GMm}{\left(\frac{r}{2}\right)^2} = m\left(\frac{r}{2}\right)\left(\frac{2\pi}{T}\right)^2 \implies T = \pi\sqrt{\frac{r^3}{2GM}}$$

Therefore, as we have reasoned, the time taken for the dust particle to reach the Earth is half of this value,

$$\frac{T}{2} = \frac{\pi}{2\sqrt{2}}\sqrt{\frac{r^3}{GM}}$$

so the answer is $\boxed{\text{(c)}}$.

Question 23

At the minimum height, the gravitational forces by the Earth and Sun are equal, so

$$\frac{GM_E m}{(h + R_E)^2} = \frac{GM_S m}{(d_{SE} - (h + R_E))^2} \implies h = \frac{d_{SE}}{1 + \sqrt{\frac{M_S}{M_E}}} - R_E \approx 2.53 \times 10^5 \text{ km}$$

i.e. the answer is $\boxed{\text{(c)}}$.

Question 24

The total energy is the sum of the kinetic and potential energies,

$$E = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

which is the negative of kinetic energy. Therefore, as resistive forces decrease the total energy, the kinetic energy increases. As a result, the radius decreases and the speed increases, so the answer is $\boxed{\text{(b)}}$.

Question 25

Transformers preserve the magnetic flux through them, so

$$B_1 A_1 = B_2 A_2 \implies \frac{B_1}{B_2} = \frac{A_2}{A_1}$$

i.e. the answer is $\boxed{\text{(c)}}$.

Question 26

A non-ideal voltmeter of internal resistance R_V can be treated as an ideal voltmeter in parallel with a resistor R_V, while a non-ideal ammeter of internal resistance R_A can be treater as an ideal ammeter in series with a resistor R_A. Hence, when the two components are connected in series with an ideal battery of emf ε, the voltmeter measures the voltage V across the internal resistance R_V, while the ammeter measures the current I through the internal resistance R_A. We can then deduce that

$$\begin{aligned} I &= \frac{\varepsilon}{R_V + R_A} \\ V &= IR_V \end{aligned}$$

By connecting a resistor in parallel with the voltmeter, the internal resistance R_V effectively decreased to a different value R'_V, while the reading on the voltmeter became $\frac{V}{2}$ and the reading on the ammeter became $4I$. Since these two equations still hold, the second equation gives us

$$\frac{V}{2} = 4IR'_V \implies R'_V = \frac{V}{8I} = \frac{R_V}{8}$$

Then, the first equation gives us

$$4I = \frac{\varepsilon}{R'_V + R_A} = \frac{\varepsilon}{\frac{R_V}{8} + R_A}$$

Dividing this with the original first equation, we finally have

$$4 = \frac{R_V + R_A}{\frac{R_V}{8} + R_A} \implies \frac{R_V}{R_A} = 6$$

so the answer is $\boxed{\text{(d)}}$.

Question 27

Since they are connected in series, the charge on the wire connecting the two capacitors is conserved and their voltages sum to that of the battery, so

$$\begin{aligned} Q_2 - Q_1 &= Q - 3Q = -2Q \\ \frac{Q_1}{C} + \frac{Q_2}{2C} &= V \end{aligned}$$

Solving, we obtain

$$Q_1 = \frac{2}{3}CV + \frac{2}{3}Q$$
$$Q_2 = \frac{2}{3}CV - \frac{4}{3}Q$$

so the answer is $\boxed{\text{(c)}}$.

Question 28

Since the electric field within the conducting shell is zero, the charge $+q$, which is at the centre of the shell's cavity, induced a charge of $-q$ uniformly distributed on the inner surface of the shell, to ensure that the electric field in the shell is zero by Gauss's law. On the other hand, the charge $+Q$ causes the outer surface of the shell with total charge $+q$ to be polarised, inducing a net positive charge away from $+Q$ and a net negative charge towards $+Q$.

The polarised charges on the outer surface distribute themselves such that the electric field within the cavity of the shell cancels that due to the charge $+Q$. As such, $+q$ is electrostatically shielded from $+Q$ and experiences no electric force from the charges on the inner surface of the shell, so it experiences no net force.

On the other hand, the charge $+Q$ experiences both the force from the charge $+q$ as well as the force from the charges on the shell. In general, these forces will not cancel, and $+Q$ experiences a net force.

Therefore, $+q$ experiences no net force but $+Q$ experiences a net force, so the answer is $\boxed{\text{(e)}}$.

Question 29

The electric field must be spherically symmetric, so taking a spherical Gaussian surface of radius r concentric to the charged sphere, by Gauss's law,

$$E \cdot 4\pi r^2 = \frac{q_{\text{enc}}}{\varepsilon_0} \implies E \propto \frac{q_{\text{enc}}}{r^2}$$

Since the charge is uniformly distributed, the enclosed charge is proportional to the enclosed volume, so

$$E \propto \frac{V_{\text{enc}}}{r^2} \propto \frac{r^3 - c^3}{r^2} = r - \frac{c^3}{r^2}$$

i.e. the answer is $\boxed{\text{(d)}}$.

Question 30

By symmetry, the electric field must point away from the opposite corner. Summing the components of the electric fields due to each charge along this direction (since field is a vector quantity),

$$E = \frac{1}{4\pi\varepsilon_0}\left(2 \cdot \frac{q}{l^2}\cos 45^\circ + \frac{q}{\left(\sqrt{2}l\right)^2}\right) = \frac{q}{4\pi\varepsilon_0 l^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

so the answer is $\boxed{\text{(d)}}$.

Question 31

This energy is simply the potential energy by definition. Simply summing the electric potential energies due to each charge (since energy is a scalar quantity),

$$U = \frac{1}{4\pi\varepsilon_0}\left(2 \cdot \frac{q^2}{l} + \frac{q^2}{\sqrt{2}l}\right) = \frac{q^2}{4\pi\varepsilon_0 l}\left(2 + \frac{\sqrt{2}}{2}\right)$$

so the answer is $\boxed{\text{(e)}}$.

Question 32

In an electromagnetic wave, the magnetic field $\mathbf{B}$ oscillates perpendicularly and in phase to the electric field $\mathbf{E}$ with magnitude $B = \frac{E}{c}$. Let the electric and magnetic fields at the charge's position at some point in time be $E\hat{\mathbf{i}}$ and $B\hat{\mathbf{j}}$. The electromagnetic wave travels in the direction of $\hat{\mathbf{E}} \times \hat{\mathbf{B}} = \hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$, so the charge travels in the direction of $-\mathbf{k}$. Then, the Lorentz force that acts on the charge is

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = q\left(E\hat{\mathbf{i}} + (-v\hat{\mathbf{k}}) \times B\hat{\mathbf{j}}\right) = q(E + vB)\hat{\mathbf{i}} = qE\left(1 + \frac{v}{c}\right)\hat{\mathbf{i}}$$

whose magnitude $F = qE\left(1 + \frac{v}{c}\right)$ is clearly maximised when the electric field is maximised. Therefore, the answer is $\boxed{\text{(b)}}$.

Question 33

By symmetry, the magnetic field at both ends of the solenoid along its axis are the same. Now, consider attaching two identical long solenoids coaxially at their ends. This forms a new solenoid where the attached ends of the original solenoids are now positioned at the centre of the solenoid, so the magnetic field at that position is $B = \mu_0 nI$. On the other hand, by superposition, this must be twice the magnetic field at the ends of the original solenoids. Therefore, the magnetic field at the end of the solenoid along the axis is $\frac{B}{2} = \frac{1}{2}\mu_0 nI$, so the answer is $\boxed{\text{(b)}}$.

Question 34

Since $h \ll l$, we can treat the plane as essentially an infinite plane. By Newton's third law, the force on the plane by the charge is the same as the force on the charge by the plane. We can determine this force by using Gauss's law on a pillbox of base area A, symmetrically placed through the plane. By symmetry, there cannot by any electric field parallel to the plane, so no electric flux passes through the curved surface of the pillbox. Furthermore, by symmetry, the electric field E must be perpendicular to and uniform along the two bases of the pillbox. Hence, by Gauss's law,

$$2EA = \frac{q_{\text{enc}}}{\varepsilon_0} = \frac{AQ}{l^2\varepsilon_0} \implies E = \frac{Q}{2l^2\varepsilon_0}$$

Therefore, the force between the charge and the plane is $F = qE$, so the plane's acceleration is

$$a = \frac{qE}{m} = \frac{qQ}{2ml^2\varepsilon_0}$$

This answer is $\boxed{\text{none of the options}}$. We believe that the options are erroneous and the intended answer is (c), which assumes that the plane is a thin conducting plane with a surface charge of Q. If this is the case, then the other surface also contains charge Q that produces the same electric field, producing a factor of 2 which results in this answer option.

Question 35

The magnetic flux increases in the left half and decreases in the right half. By Lenz's law, the left half induces an anticlockwise current in itself and the right half induces a clockwise current in itself, which both result in an upward current through the rod. By Faraday's law, the magnitude of the induced emf in each half is

$$\varepsilon = \left|\frac{\mathrm{d}\mathbf{\Phi}_B}{\mathrm{d}t}\right| = B\left|\frac{\mathrm{d}A}{\mathrm{d}t}\right| = BLv$$

so the total current in the rod is

$$I = 2 \cdot \frac{\varepsilon}{R} = \frac{2BLv}{R}$$

The magnitude of the force needed to move the rod at constant speed v should balance that of the magnetic force on the rod. Since the current is perpendicular to the magnetic field, this force is then

$$F = ILB = \frac{2B^2L^2v}{R}$$

so the answer is $\boxed{\text{(d)}}$.

Question 36

Let the weight of each magnet be W and the force between two disc magnets be $F = \alpha r^{-n}$ where α is a constant. The magnet in the middle is repelled by both other magnets, while the magnet above is repelled by the middle magnet but attracted to the bottom magnet. Balancing forces on both of these magnets,

$$\alpha h_1^{-n} = \alpha h_2^{-n} + W$$
$$\alpha h_2^{-n} = \alpha (h_1 + h_2)^{-n} + W$$

Subtracting both equations,

$$\alpha\left(h_1^{-n} - h_2^{-n}\right) = \alpha\left(h_2^{-n} - (h_1+h_2)^{-n}\right) \implies \left(\frac{h_2}{h_1}\right)^n + \left(\frac{h_2}{h_1+h_2}\right)^n = 2$$

Substituting the values given, we get the equation $\left(\frac{11}{9}\right)^n + \left(\frac{11}{20}\right)^n = 2$. We can do a numerical check to find that $n \approx 3$ is the closest approximation to the (non-zero) solution, so the answer is $\boxed{\text{(c)}}$.

Question 37

Consider only injecting current I into node 1. By symmetry, current $\frac{I}{3}$ flows outwards into each adjacent resistor, so current $\frac{I}{3}$ flows from node 1 to 2. Likewise, consider only extracting current I from node 2. By symmetry, current $\frac{I}{3}$ flows inwards from each adjacent resistor, so current $\frac{I}{3}$ again flows from node 1 to node 2.

Now, consider both injecting current I into node 1 and extracting current I from node 2. By the principle of superposition, the currents in each resistor are the sum of their individual currents in each of the previous scenarios. Hence, current $I_R = \frac{I}{3} + \frac{I}{3} = \frac{2I}{3}$ flows from node 1 to node 2 in this scenario, so the potential difference between the two nodes is $\Delta V = I_R R$.

Finally, considering the entire grid as an effective resistor R_{eff}, considering the potential difference again gives

$$\Delta V = I_R R = I R_{\text{eff}} \implies R_{\text{eff}} = \frac{I_R}{I} R = \frac{2}{3} R$$

so the answer is $\boxed{\text{(e)}}$.

Question 38

The central bright fringe occurs when the difference in the optical path lengths between the two light rays reaching its position is zero. This occurs when both rays are parallel to the incident rays, so the line from the central maximum to the slits must make the same angle θ from the reference axis. Therefore, the distance of the central maximum from the reference axis is simply $x = D\tan\theta$, so the answer is $\boxed{\text{(c)}}$.

Question 39

Let the phase of the solid wave be 0 and dictate the phase to increase when it moves to the right. Then the phase of the dotted wave is $\frac{3\pi}{2}$ and decreases at the same rate when it moves to the left. For constructive interference, their phases must be the same, so the phase change $\Delta\phi_\text{c}$ is

$$\Delta\phi_\text{c} = \frac{3\pi}{2} - \Delta\phi_\text{c} \implies \Delta\phi_\text{c} = \frac{3\pi}{4}$$

so the time when it achieves constructive interference is

$$T_\text{c} = \frac{\Delta\phi_\text{c}}{2\pi}T = \frac{3}{8}T$$

For destructive interference, their phases must be π apart. This first occurs when the phase change $\Delta\phi_\text{d}$ is

$$\Delta\phi_\text{d} + \pi = \frac{3\pi}{2} - \Delta\phi_\text{d} \implies \Delta\rho_\text{d} = \frac{\pi}{4}$$

so the time when it achieves destructive interference is

$$T_\text{d} = \frac{\Delta\phi_\text{d}}{2\pi}T = \frac{1}{8}T$$

Therefore, the answer is $\boxed{\text{(b)}}$.

Question 40

The distance v_1 of the object's intermediate image to lens 1 is

$$\frac{1}{u_1} + \frac{1}{v_1} = \frac{1}{f} \implies v_1 = \left(\frac{1}{f} - \frac{1}{3f}\right)^{-1} = \frac{3}{2}f$$

In order to form a real image on the screen, the intermediate image (which acts as the object for lens 2) must be further away from lens 2 than the focal point. Therefore, it has a greater distance $d - \frac{3}{2}f$ to lens 2 than the focal length f, so at the minimum distance,

$$d - \frac{3}{2}f = f \implies d = \frac{5}{2}f$$

i.e. the answer is $\boxed{\text{(d)}}$.

Question 41

The dominant pitch heard is the fundamental frequency. Since the tube in which the air vibrates in is closed at one end (the water surface) and open and the other end (the mouth), the fundamental frequency occurs when the wavelength is four times the length of the tube,

$$\lambda = 4\left(h - \frac{h}{4}\right)3 = h$$

Therefore, the frequency f he hears is

$$c = f\lambda \implies f = \frac{c}{3h} = 390 \text{ Hz}$$

The closest frequency to this value is G4 (392 Hz), so the answer is $\boxed{\text{(a)}}$.

Question 42

On this hyperbola, since it passes through $(0, \frac{a}{2})$, the difference in the optical path lengths between the rays that reach these two points constructively interfere. This difference is

$$\Delta x = \left(\frac{a}{2} - (-a)\right) - \left(a - \frac{a}{2}\right) = a$$

Everywhere on this hyperbola must correspond to the same phase difference, so the difference in the optical path lengths of the rays that intersect at this hyperbola are the same everywhere. Therefore, equating this value to the difference between the path lengths at (x, a),

$$\Delta x = a = \sqrt{x^2 + (a - (-a))^2} - x = \sqrt{x^2 + 4a^2} - x$$

Isolating the square root and squaring both sides, we obtain

$$x^2 + 4a^2 = (a + x)^2 = x^2 + 2ax + a^2 \implies x = \frac{3a}{2}$$

so the answer is $\boxed{\text{(c)}}$.

Question 43

Some geometrical consideration shows that the angle of incidence of the ray at the sides and at the ends of the cylinder is constant, and has values $\theta_\mathrm{s} = 90^\circ - \theta$ and $\theta_\mathrm{e} = \theta$ respectively. Both of these values have to be greater than the critical angle $\theta_\mathrm{c} = \sin^{-1}\left(\frac{1}{n}\right) = 42^\circ$ for total internal reflection to occur, so we have

$$\begin{cases} 90^\circ - \theta > \theta_\mathrm{c} \\ \theta > \theta_\mathrm{c} \end{cases} \implies 42^\circ < \theta < 48^\circ$$

i.e. the answer is $\boxed{\text{(e)}}$.

Question 44

Since energy is conserved, the total power P carried along a wavefront must be constant. Hence, the intensity I must be inversely proportional to the area of the wavefront,

$$I(r) = \frac{P}{4\pi r^2} \propto \frac{1}{r^2}$$

The intensity is also proportional to the square of the amplitude, so we have

$$A(r) \propto \sqrt{I(r)} \propto \frac{1}{r}$$

The phase of the wave is also the same everywhere along a wavefront, so the argument to the cosine term must be constant. Since r increases as t increases, this requires $\omega < 0$. Furthermore, the speed of the wave is c, so

$$c = \frac{|\omega|}{k} \implies \omega = -ck$$

i.e. the answer is $\boxed{\text{(b)}}$.

Question 45

At any point on the $P - V$ diagram, since the gas is monatomic and ideal, the internal energy is

$$U = \frac{3}{2}nRT = \frac{3}{2}PV$$

We also know that the work done on the system in each of the processes is the negative of the area under the curve in the $P - V$ diagram, since the gas expands. We can then use the first law of thermodynamics to calculate the net heat in each of the respective processes,

$$Q_{AB} = \Delta U_{AB} - W_{\text{on},AB} = \frac{3}{2}(3P_0 \cdot 3V_0 - 5P_0 \cdot V_0) + \frac{1}{2}(5P_0 + 3P_0)(3V_0 - V_0) = 14P_0V_0$$
$$Q_{BC} = \Delta U_{BC} - W_{\text{on},BC} = \frac{3}{2}(P_0 \cdot 5V_0 - 3P_0 \cdot 3V_0) + \frac{1}{2}(3P_0 + P_0)(5V_0 - 3V_0) = -2P_0V_0$$

Therefore, heat is absorbed during the process $A \to B$ while heat is released during the process $B \to C$, so the answer is $\boxed{\text{(b)}}$.

Question 46

(a) This statement is false; the second law of thermodynamics has nothing to do with this.

(b) This statement is false; the correct statement for the conservation of energy is $100 \text{ J} = 80 \text{ J} + 20 \text{ J}$.

(c) This statement is false; that is how heaters and refrigerators operate, not engines.

(d) This statement is true, as the claimed efficiency of this engine is $\eta = \frac{80 \text{ J}}{100 \text{ J}} = 80\%$ but the Carnot efficiency is only $\eta_C = \frac{283 \text{ K}}{373 \text{ K}} = 76\%$.

(e) This statement is false, as we have just shown.

Therefore, the answer is $\boxed{\text{(d)}}$.

Question 47

Consider a molecule moving at speed $v_\perp$ perpendicular to a pair of opposite walls, separated by distance L. Then the amount of time between collisions with one of the walls is $t = \frac{2L}{v_\perp}$ (since the molecule needs to traverse back and forth), so the collision rate of this one molecule is

$$\nu_0 = \frac{v_\perp}{2L}$$

Over all N molecules, we can make a very crude approximation for the average value of $v_\perp$ as the root-mean-square speed v_{rms} of the molecules, as these two quantities will not differ by more than an order of magnitude. The value of this speed is

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

where T and M are the temperature and molar mass respectively. Furthermore, by the ideal gas law, the total number of molecules is

$$PV = Nk_BT \implies N = \frac{PL^3}{k_BT}$$

where P is the pressure. Finally, since the box has six faces, our crude approximation to the rate of collisions between the molecules and the walls is

$$\nu \sim 6N \cdot \frac{v_{\text{rms}}}{2L} = 3\sqrt{3}\frac{PL^2}{k_B}\sqrt{\frac{R}{MT}} \sim 10^{28} \text{ s}^{-1}$$

so the answer is $\boxed{\text{(d)}}$.

Question 48

Let the number of moles in the first chamber be n. The two chambers contain equal masses of gas X, but the chamber containing gas of the form X_2 has twice the molar mass, so it must contain half the number of moles in the first chamber, $\frac{n}{2}$.

As no heat is exchanged to the surroundings, the total internal energy of the gas system is conserved. Therefore, we can solve for the final temperature T of the final mixture as

$$\frac{3}{2}nRT + \frac{5}{2}\left(\frac{n}{2}\right)nRT = \frac{3}{2}nRT_1 + \frac{5}{2}nRT_2 \implies T = \frac{6T_1 + 5T_2}{11}$$

so the answer is $\boxed{\text{(e)}}$.

Question 49

The Earth attains its surface temperature when the heat that is radiates balances the heat received from radiation by the sun. By the Stefan-Boltzmann law, the power radiated by Earth is

$$P = \sigma A_E T^4 \propto T^4$$

where $A_E = 4\pi R_E^2$ is the surface area of the Earth. Meanwhile, the sun radiates a constant power P_S, so its intensity when it reaches the Earth is

$$I = \frac{P_S}{4\pi r^2} \propto \frac{1}{r^2}$$

where r is the Sun-Earth distance. Then the power received by Earth is

$$P = IA_{\text{eff}} \propto \frac{1}{r^2}$$

where $A_{\text{eff}} = \pi R_E^2$ is the effective surface area of the Earth. These two quantities are equal, so we must have

$$T^4 \propto \frac{1}{r^2} \implies T \propto \frac{1}{\sqrt{r}}$$

Therefore, the new average surface temperature T' of the Earth is

$$\frac{T'}{T} = \sqrt{\frac{r}{r'}} \implies T' = \frac{T}{\sqrt{0.90}} = 31^\circ\text{C}$$

so the answer is $\boxed{\text{(c)}}$.

Question 50

As the tube and droplet are perfect thermal insulators and the tube is quickly brought vertical, no heat is exchange and the compression of the gas enclosed is adiabatic, with adiabatic index $\gamma = \frac{5}{3}$. Initially, the droplet is stationary, so the pressure of the gas enclosed is equal to the atmospheric pressure. Afterwards, balancing forces on the droplet,

$$PA = P_0A + mg \implies P = P_0 + \frac{mg}{A}$$

Using the fact that the compression is adiabatic,

$$\left(P_0 + \frac{mg}{A}\right)\left(\frac{3}{4}AL\right)^\gamma = P_0(AL)^\gamma \implies m = \left(\left(\frac{4}{3}\right)^\gamma - 1\right)\frac{P_0A}{g} = 0.62\frac{P_0A}{g}$$

so the answer is $\boxed{\text{(b)}}$.