2026 Selection Test

for the International Physics Olympiad (IPhO), Asian Physics Olympiad (APhO) and International Nuclear Science Olympiad (INSO)

Name: ____

a. This is a **4 hour** test. Attempt all questions. The maximum total score is **100**; marks allocated for each question part are indicated in square brackets.

b. Check that there are a total of **14 printed pages** (*including* this cover page). The table of physical constants will be handed out to you separately.

c. Begin your answer for each question on a **fresh sheet of paper**, and present your working and answers clearly. Your answer sheets should be sorted according to the order of the questions.

d. Write your name on the **top right hand corner of every answer sheet** you submit.

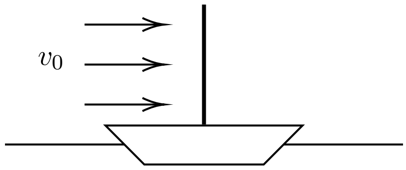
e. Submit this question paper and all your working sheets. No paper, whether used or unused, may be taken out of this examination room.

f. You may use a standard (non-programmable) scientific **calculator** in accordance with the statutes of the International Physics Olympiad.

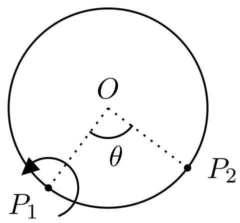
g. No books or documents relevant to the test may be brought into the examination room.

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	9	9	4	10	10	10	7	8	15	18	100
Score:											

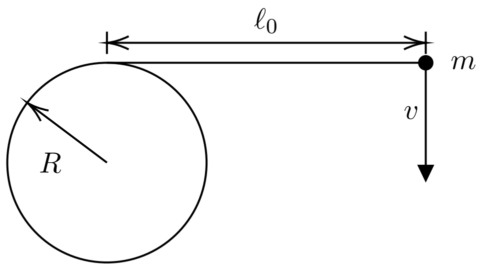
1. (a) A sailboat is moving in stationary water. A wind with uniform density and horizontal speed v_0 (relative to the ground) is blowing perpendicularly to the surface of the sail. Find the speed of the sailboat where the wind exerts maximum power. [2]



(b) A circular uniform disk is initially rotating about point P_1 on its circumference at a constant angular velocity ω_1. If point P_1 is suddenly released and another point P_2 on the circumference is simultaneously fixed in place, what is the new angular velocity of the disk ω_2 about P_2. Take angle $\angle P_1OP_2$ to be θ. [3]



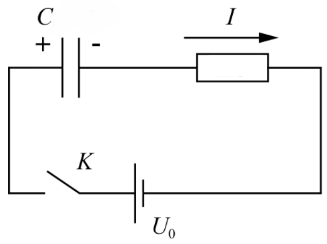
(c) A cylindrical disk of radius R lies flat on a smooth horizontal surface and is fixed in place. An inextensible thread is tightly wound on the disk, and the free end is attached to a small puck with mass m. The length of the free part of the thread is ℓ_0. The puck is initially given a velocity v perpendicular to the thread. Assuming the thread can only withstand some maximum tension T, explain whether the puck be able to reach the disk. If yes, find the time taken for the puck to hit the disk. If not, find the time taken for the thread to break in terms of T and other relevant constants. [4]

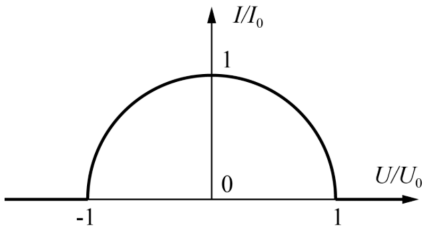


2. In the figure below, we have an electrical circuit consisting of a DC voltage source U_0, capacitor C, switch K and a nonlinear element. The current-voltage characteristic of the nonlinear element is as follows:

$$\left(\frac{I}{I_0}\right)^2 + \left(\frac{U}{U_0}\right)^2 = 1$$

The equation holds when $I > 0$ and for $|U| \geq U_0$, $I = 0$. I_0 and U_0 are known.





(a) Consider the above circuit without the nonlinear element first. The capacitor is initially uncharged; at $t = 0$, the switch is closed to charge the capacitor. What is the final voltage across the capacitor? [1]

(b) Now, with the capacitor charged to the voltage in (a) and the switch open, the nonlinear element is reintroduced and the switch is again closed at $t = 0$. What is the [4]

a. time taken for the second charging process, and

b. the final voltage across the capacitor?

Justify your answers and show all working clearly.

(c) Find the maximum rate of change of capacitor energy. At what time t does this occur? [2]

(d) Now consider the same nonlinear circuit, but the capacitor has a *very small positive charge* $\delta \ll CU$ before the switch is closed. Find the time taken in this charging process and the final voltage across the capacitor. Explain why it can be said that this process is a transition from unstable to stable equilibrium. [2]

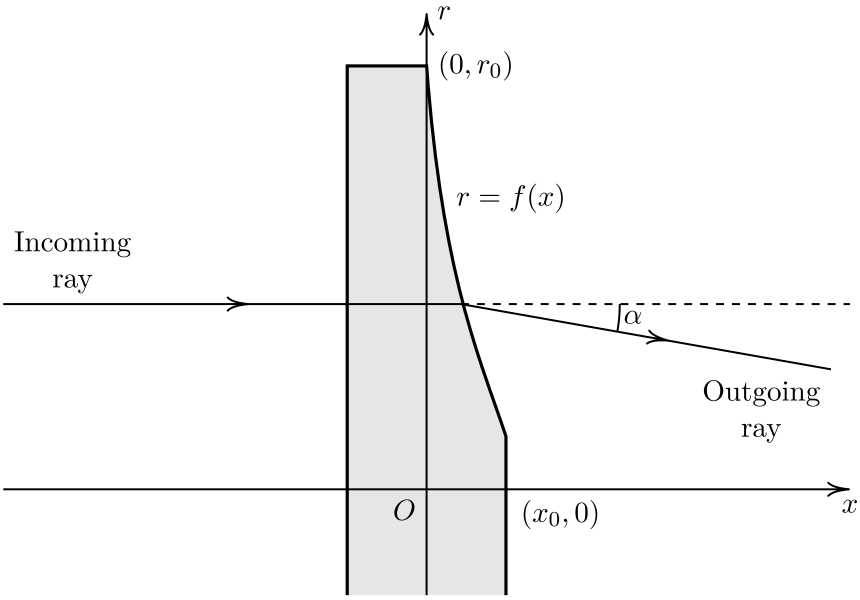
3. In General Relativity, light rays can get deflected by massive bodies. For a spherically-symmetric body, if the undisturbed motion of the ray passes the centre of the body at a minimum distance of r (the impact parameter), the angular deflection (in radians) is given by: [4]

$$\alpha = \frac{4GM}{rc^2}$$

for $\alpha \ll 1$ rad. We aim to construct a lens out of plastic with refractive index n that simulates this effect. The lens is constructed using the volume of revolution of a function $r = f(x)$ about the x-axis. We want to choose $f(x)$ such that the angle of deflection at an impact parameter of r is given by:

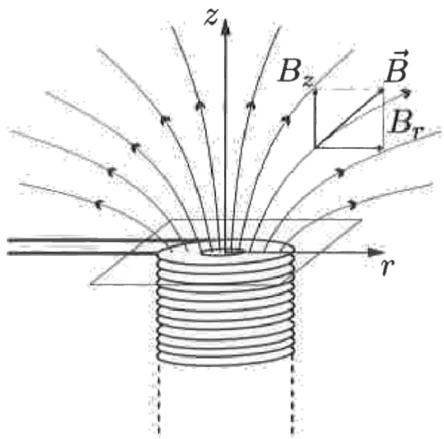
$$\alpha = \frac{s}{r}$$

where s is some constant. You may use small angle approximation for the angle of incidence and α, and assume that air has refractive index 1.



Determine $f(x)$ for $x \in [0, x_0]$ shown in the diagram above in terms of x, n, s and r_0.

4. Optical microscopes use optical lenses to bend light and form images. Electron microscopes on the other hand rely on electromagnetic fields to bend electrons. We will explore the properties of a magnetic lens in the following question. For all parts below we will work in cylindrical coordinates (r, θ, z) with the origin coninciding with the center of the top face of the magnet.



We can use a cylindrically symmetric magnetic field as a lens. Near the central axis of the magnet (small r), the z component of the field can be approximated by

$$B_z(z) = \frac{B_0}{1 + (\frac{z}{a})^2},$$

where a is some constant.

(a) Show that the radial magnetic field B_r near the central axis is given by [1]

$$B_r = -\frac{r}{2}\frac{dB_z}{dz}$$

(b) By considering the equation of motion for an electron with mass m and charge $-e$ originating far away from the lens with some initial position (r_0, θ_0, z_0), show that the angular position of the electron is governed by [4]

$$\dot{\theta} = \frac{e}{2m} B_z$$

You may assume that the electrons are paraxial (r_0 is small) and that the initial speed v of the electron is almost entirely in z direction ($v \approx v_z$) at all times.

(c) From the equation of motion in the radial direction, derive the following equation relating z and r for the trajectory of the particle: [2]

$$\frac{d^2y}{dx^2} = -\frac{k^2}{(1+x^2)^2} y$$

where $y = \frac{r}{a}$, $x = \frac{z}{a}$ and k is to be determined in terms of the electron's initial kinetic energy, E, its mass, m, and other constants.

(d) The general solution to the equation can be found using the substitutions $x = \frac{z}{a} = \cot(\phi)$ and $y = \frac{r}{a}$: [3]

$$y(\phi) = C_1 \frac{\sin(\omega\phi)}{\sin\phi} + C_2 \frac{\cos(\omega\phi)}{\sin\phi}, \quad \text{where } \omega = \sqrt{1+k^2}$$

where C_1 and C_2 depend on the initial direction and position of the electron. If a point source of electrons emitting electrons in all directions is at some point $P_0(y_0, \phi_0)$, determine the ϕ values (ϕ_n) where the emitted electrons converge. Also determine the minimum k such that two images will be formed for any ϕ_0.

5. In this problem, we will study the thermodynamics of an unusual heat engine, known as an Archibald rubber band heat engine.

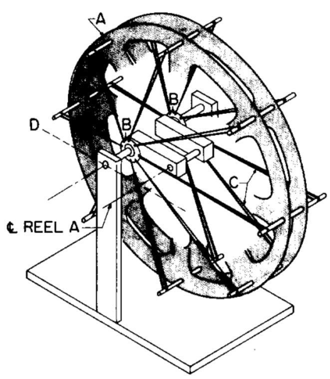
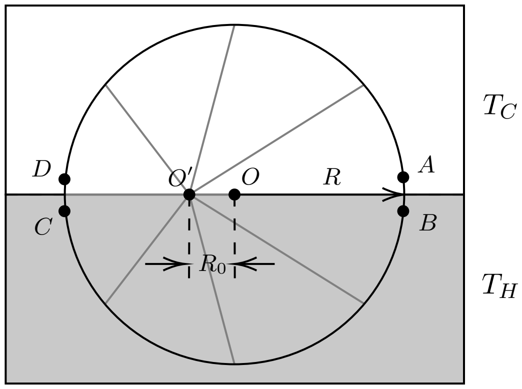


Fig. 1. Non-scale drawing of Archibald rubber-band heat engine constructed for classroom demonstrations. The only parts which rotate are the 14 inch reel, A, shown with the center portion cut away, the Teflon washers, B, and the rubber bands, C. Dashed line D indicates the water level normally used when operating the engine. A Teflon bushing, not shown, was inserted into the center of movie reel, A, to reduce friction.

This heat engine consists of a series of rubber bands with one end attached to the circumference of a wheel with radius R, and the other end fastened to a frictionless bearing, whose center is offset from the wheel's central axis of rotation by a distance R_0. Both the wheel's axis and rubber band axis are fixed and do not move.

A "heat engine" can be constructed by submerging the bottom half of the assembly in hot water with uniform temperature T_H, which causes the wheel and attached bands to rotate. The top half remains at ambient temperature T_C. A clearer diagram is shown below. The rubber bands are connected to O', while the wheel rotates about O.



(a) Given the set up above where the rubber band axis is offset to the left by R_0 and that stretched rubber contracts when heated, explain whether the wheel rotates clockwise or counterclockwise. [1]

The thermodynamic relation for a rubber band is given by

$$dU = TdS + \tau dL$$

where T is the temperature, τ is the tension, U is the internal energy, S is the entropy, and L is the length of the rubber band. You may assume that just like an ideal gas, U can be expressed as a function of only the temperature T.

(b) By referencing the Maxwell relation for a typical P, V, T system: [1]

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V,$$

derive an equivalent Maxwell relation for the rubber band with a suitable substitution.

We assume that that change in tension in the rubber band is small in one cycle and may be expanded linearly about the tension at length R and average temperature $\bar{T} = (T_C + T_H)/2$ as

$$\tau(L, T) = \tau(R, \bar{T}) + \rho(L - R) + \sigma(T - \bar{T})$$

where $\rho = (\partial\tau/\partial L)_T$ and $\sigma = (\partial\tau/\partial T)_L$ both evaluated at $(R, \bar{T})$ are constants.

(c) By considering the entropy S as a state function of the temperature T and length L of the rubber band, show clearly that for a reversible process, we have [2]

$$dQ = C_L\, dT - T\sigma\, dL$$

where C_L is defined as the heat capacity at constant length.

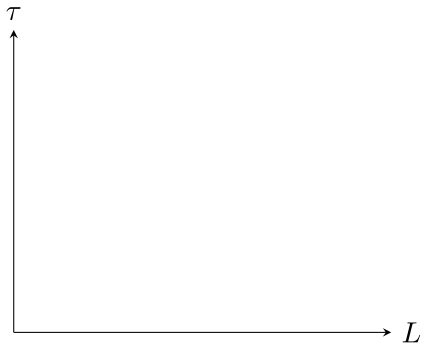
Hint: For a multivariate function $f(x, y)$, we may write its differential

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

You may also use the Maxwell relation derived in (b).

(d) We now consider the thermodynamic cycle undergone by a rubber band on the wheel during a full rotation. We consider the state of the rubber band as it rotates through 4 locations A, B, C and D shown in the second diagram above. Sketch a τ-L diagram connecting A, B, C, D (on your answer sheet) and use arrows to denote the direction of the thermodynamic cycle as the wheel rotates in the direction given by your answer [3] in (a). Label the horizontal coordinates of A, B, C, D in terms of R and R_0. You may also use the linear approximation of τ about $(R, \bar{T})$ if necessary.

Hint: You may assume two of the processes to be "isochoric" and two of the processes to be isothermal, but which?



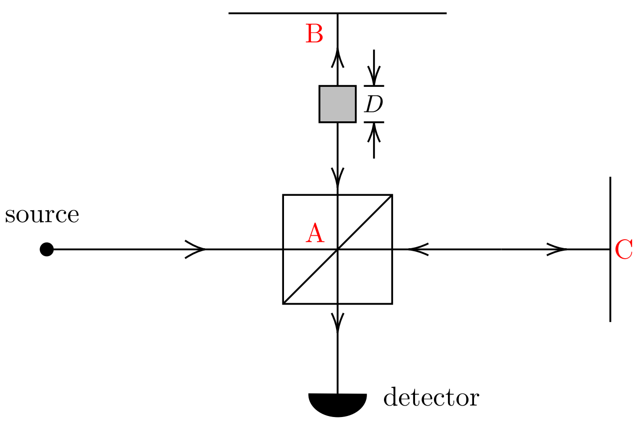
(e) By considering the heat cycle in (d), calculate the thermodynamic efficiency η of this engine in terms of T_C, T_H, σ and R_0. You may use the simplification [3]

$$\alpha = \int_{T_C}^{T_H} C_L \, dT$$

in your answer without explicitly evaluating the integral.

6. In this question, we will consider variations of the Michelson-Morley interferometer.

(a) In the interferometer below, an ideal beam splitter is placed at A while two perfectly reflective mirrors are placed at B and C, with $|AB|$ and $|AC|$ being equidistant. To mesaure the refractive index of a gas such as helium, we insert a hollow glass cell of width $D = 0.10$ m and negligible thickness into one path of the interferometer and evacuate it. Monochromatic light of wavelength λ is shone into the interferometer. [2]



Helium gas with refractive index n is then slowly added to the cell until the pressure reaches atmospheric pressure. As this is done, the intensity of the light at the detector will vary. We then carefully count the number of times the intensity varies from maximum to minimum and back to maximum. If $\lambda = 633$ nm (from a helium-neon laser) and there are $k = 11$ cycles back to maximum intensity, find the numerical value of $n-1$ at atmospheric pressure. The interferometer is placed in the x-y plane and you may neglect effects of gravity.

(b) Suppose the tiny gas chamber is now moving at speed v from A towards B, find the phase difference $|\Delta\phi|$ between the light from the two paths meeting at the detector in terms of the refractive index n, wavelength λ, and other relevant constants. [4]

(c) Now, we consider a different Michelson-Morley interferometer set up in the vertical plane, with the top mirror at B removed. A beam of neutrons, each having mass m, is fired from the left. The splitter placed at A causes half of it to take the horizontal path with distance L to a mirror at C and back, while the other half takes the vertical path against gravity to some turning point B' and back. The initial kinetic energy of each neutron is $\varepsilon = \alpha mgL$. [3]

By treating the neutrons as non-relativistic de Broglie waves, derive the expression for the phase difference $|\Delta\phi|$ when the two neutron beams meet at the detector in terms of α, m, g, L, and $\hbar$. **Effects of gravity are not negligible**.

(d) Qualitatively sketch the graph of intensity I measured at the detector against α for $0 \le \alpha \le 6$ assuming the number of neutrons emitted per unit time is the same. [1]

7. In this question, we will learn more about the interaction between atoms and light. Let us consider a cavity of volume V held at constant temperature T. Within this cavity is a "photon gas" in thermal equilibrium with the walls. We model these photons as ideal gas particles with a mean energy per photon of $\hbar\langle\omega\rangle$.

(a) Using arguments from Kinetic Particle Theory or otherwise, show that the power incident per unit area of cavity wall $\langle I\rangle$, due to the photons, can be written as [2]

$$\langle I\rangle = \frac{c}{4}\langle U\rangle$$

where $\langle U\rangle$ is the energy density of the photon gas and c is the speed of light.

For a perfect blackbody at thermodynamic equilibrium at temperature T, the power incident per unit area per unit angular frequency $I(\omega)$ is equal to the power emitted per unit area per unit angular frequency, given by the famous Planck's Law

$$I(\omega) = \frac{\hbar\omega^3}{4\pi^2c^2}\frac{1}{e^{\hbar\omega/(k_BT)}-1} = \frac{c}{4}U(\omega)$$

We now introduce a collection of atoms into the cavity. Each atom has a ground state $|a\rangle$ with energy E_a and an excited state $|b\rangle$ with energy E_b, separated by $\Delta E = E_b - E_a = \hbar\Delta\omega$. The number of atoms in each of these states are N_a and N_b respectively. The atoms then interact with the photon gas, and Einstein identified three fundamental processes occuring:

a. Spontaneous emission: Atom naturally decays from $|b\rangle \to |a\rangle$ with rate k_1, emitting a photon.
b. Stimulated emission: An incident photon triggers an atom to decay from $|b\rangle \to |a\rangle$ with rate k_2, emitting a second, identical photon.
c. Stimulated absorption: An atom absorbs an incident photon gets excited from $|a\rangle \to |b\rangle$ with rate k_3.

(b) The actual rates used by Einstein when he tackled this problem in 1917 are αN_b, $\beta U(\Delta\omega)N_a$, $\gamma U(\Delta\omega)N_b$, where α, β and γ are new numerical constants he defined. Match these to the rate $k_{1,2,3}$ corresponding to the 3 processes above and justify your choices fully with physics. [1]

(c) Given that the atoms are in thermal equilibrium with the bath of photons and by considering $\dot{N}_a$ and $\dot{N}_b$, show the following relationships [4]

$$\beta = \gamma \quad \text{and} \quad \alpha = \frac{\hbar \Delta\omega^3}{\pi^2 c^3}\beta$$

8. (a) Inverse beta decay is an important reaction in neutrino detectors. One version of this decay occurs when an electron antineutrino interacts with a proton to produce a neutron and a positron: [4]

$$\bar{\nu}_e + \mathrm{p} \longrightarrow \mathrm{n} + \mathrm{e}^+$$

Assuming the proton is at rest in the lab frame, determine the minimum neutrino energy in the lab frame required for this reaction to take place. Leave your answer in terms of the relevant rest masses of the proton m_p, neutron m_n, and electron m_e. The mass of the neutrino is negligible.

(b) Bob decides to take a spaceship with constant proper acceleration g. In the lab frame, his position x as a function of his proper time τ is given by: [4]

$$x(\tau) = \frac{c^2}{g}\cosh\frac{g\tau}{c}$$

where $\cosh x = \frac{e^x + e^{-x}}{2}$.

Alice, afraid to leave Bob, attaches herself behind Bob's spaceship with a rope of constant proper length L. Determine the proper acceleration g' experienced by Alice.

Hint: Proper time τ is always time measured in the frame of the moving body, while coordinate time t can be time measured in any frame. The proper distance Δs in any frame between two fixed events is given by:

$$\Delta s^2 = \Delta x^2 - c^2 \Delta t^2$$

The following identities may be helpful

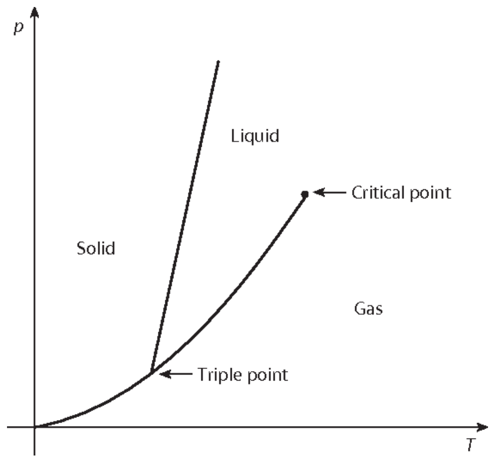
$$\frac{d}{dx}\sinh x = \cosh x, \quad \frac{d}{dx}\cosh x = \sinh x, \quad \frac{\sinh x}{\cosh x} = \tanh x$$

9. The temperature in which a thermodynamic phase transition happens can be plotted against pressure in a $P(T)$ graph. This graph is called a coexistence curve and it is determined by the Clausius-Clapeyron equation:

$$\frac{dP}{dT} = \frac{L}{T(v_2 - v_1)}$$

where L is the latent heat per mole, and v_1 and v_2 are the volumes per mole of the material in each of the phases. As an illustration, the coexistence curve of a water is shown below.

(a) First, consider a simple phase transition – a liquid-gas phase transition. You are given that the point (P_0, T_0) lies on the coexistence curve. Determine the equation of $P(T)$, making (and justifying) any relevant approximations about liquids and gases. You may assume the latent heat of vaporisation L is temperature-independent. *To not confuse the ideal gas constant with the radius (in later parts), you may write the former as* R_{g}. [2]



From now on, we shall let $P_{\text{sat}}(T)$ denote the saturation pressure; that is, the pressure that lies on the coexistence curve (the $P(T)$ found previously) for a given temperature. Consider a container held at a fixed external pressure P_{ext}. By definition, ordinary boiling occurs at a temperature T_{b}, whereby

$$P_{\text{sat}}(T_{\text{b}}) = P_{\text{ext}}$$

In the following parts, we shall study the dynamics of nucleation, referring to the initial formation of a microscopic vapour bubble inside a liquid.

For the following parts, assume that:

i. The liquid and vapour are both at a temperature T.
ii. The vapour inside the microscopic bubbles can be treated as having a pressure $P_{\text{sat}}(T)$.
iii. The surrounding bulk liquid is at a constant pressure P_{ext}.
iv. The surface tension σ is constant.
v. Gravity can be neglected for pressure variations in the liquid.
vi. The formation of bubbles is a fully reversible process.

We define the driving pressure for vapour-bubble formation as follows:

$$\Delta P(T) = P_{\text{sat}}(T) - P_{\text{ext}} \quad (T > T_{\text{b}})$$

(b) For the case $T > T_{\text{b}}$, vapour bubbles can form in the liquid. Gibbs free-energy is a thermodynamic potential measuring the maximum reversible work by a thermodynamic system at constant pressure and temperature; we can use it to determine the spontaneity and energetic feasibility of phase transitions. The Gibbs free-energy can be given by [3]

$$G_{\text{b}}(R) = U + PV - TS$$

where U is internal energy, P is pressure, V is volume and S is entropy. Find an expression for the free-energy change $\Delta G_{\text{b}}(R)$ when forming a spherical bubble of vapour of radius R. You must explain the physical meaning and the signs of each term in your answer.

(c) Determine expressions for the critical radius $R_{\text{c,b}}$ where $\Delta G_{\text{b}}(R)$ is maximum, and the corresponding nucleation barrier height (maximum free-energy barrier) ΔG^*_{b}. [1]

Continue working in the regime $T > T_{\mathrm{b}}$, so that bubble nucleation is relevant. You may treat ΔP as roughly constant during a short time interval. A vapour bubble of radius $R\left(t\right)$ expands in an incompressible, ideal liquid of mass density ρ. Neglect viscous and dissipative effects, and assume that the container of liquid is large.

Assume the liquid around the bubble to be incompressible, and that the resulting liquid velocity field $\mathbf{u} = u\left(r,t\right)\hat{\mathbf{r}}$ is purely radial, where $r > R\left(t\right)$ is the distance from the center of the bubble, outside the bubble.

(d) Derive an expression for $u\left(r,t\right)$, in terms of $R\left(t\right), \dot{R}\left(t\right)$ and r. Show your working clearly. [2]

Hint: If needed, the continuity equation is $\nabla \cdot \mathbf{v} = 0$*, where* $\mathbf{v}$ *is the velocity field of the liquid. For a purely radial vector field* $\mathbf{v} = v_r\hat{\mathbf{r}}$*, the divergence in spherical coordinates is given by* $\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right)$*.*

(e) Hence, show that [4]

$$\rho f_1\left(R, \dot{R}, \ddot{R}\right) = \Delta P - f_2\left(R\right)$$

for some functions $f_1\left(R, \dot{R}, \ddot{R}\right)$ and $f_2\left(R\right)$ that depend on the given parameters. Find the functions $f_1\left(R, \dot{R}, \ddot{R}\right)$ and $f_2\left(R\right)$.

Let T_{R} be the temperature where a bubble of radius R remains in mechanical equilibrium, under the conditions and assumptions of the previous parts. Clearly, $T_{\mathrm{R}} \neq T_{\mathrm{b}}$. We denote $\Delta T = T_{\mathrm{R}} - T_{\mathrm{b}}$.

(f) Assuming that $\Delta T \ll T_{\mathrm{b}}$, determine an expression for ΔT. [3]

10. We know that for all solutions to the linear wave equation, the principle of superposition applies. Examples include electromagnetic waves and waves on a string.

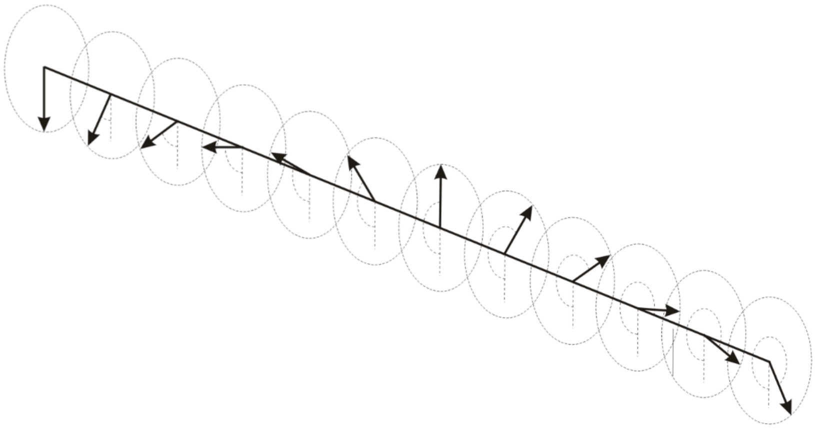
Dispersion relations describe the relation between angular velocity $\omega(k)$ and wavenumber k, where phase velocity can be obtained as $v_p = \frac{\omega}{k}$. Each wave component has its own phase velocity, propagating independently of each other.

However, in nonlinear dynamics, this is no longer the case. Interfering wave packets interact, leading to nontrivial behaviour; in dispersive media, nonlinear interactions can even preserve wave packets and oppose dispersion. These wave packets are called **solitons**.

To investigate solitons, we will look at a simple mechanical system. Consider a long taut horizontal string, to which at equal intervals b, identical spokes arranged vertically are attached. These spokes can be considered as pendulums swinging in the plane perpendicular to the string axis. The weight of each spoke is m, the distance from the center of mass of the spoke to the string is d, and the moment of inertia relative to the axis passing through the string is I. When one spoke deviates from the neighboring one by an angle $\Delta\phi$, the string creates a restoring torsional torque $\tau = -K\Delta\phi$.

It can be assumed that the characteristic size of the soliton $\lambda \gg b$. Hence, consider that the angular coordinate at some point x along the string is given by a continuous function $\phi(x,t)$; $\phi = 0$ is defined at the position where the pendulum points vertically downwards, and increases as the pendulum rotates anticlockwise.

Take the level of a line passing through the lowest possible positions of the center of mass of the spokes as zero of the potential energy in the gravitational field.



(a) Assuming the absence of gravity first: obtain a differential equation for $\phi(x,t)$ of the following form [2]

$$\frac{\partial^2\phi(x,t)}{\partial t^2} = \alpha\frac{\partial^2\phi(x,t)}{\partial x^2}$$

where α is some constant.

(b) Using the substitution $\phi(x,t) = f(x \pm vt)$, find the speed of linear wave propagation v_0 for the scenario in (a). Show your substitution clearly. [1]

(c) Now, considering gravity, obtain a differential equation for $\phi(x,t)$ of the following form [1]

$$\beta\frac{\partial^2\phi(x,t)}{\partial t^2} - \gamma\frac{\partial^2\phi(x,t)}{\partial x^2} + \delta\sin\phi(x,t) = 0$$

where β, γ, and δ are constants to determined. DO NOT use the small angle approximation.

(d) Deduce the **maximum** speed of wave propagation $v_{\max}$ for the scenario in (c). Justify your answer. [1]

The equation given in (c), ignoring coefficients, is called the **Sine-Gordon equation**; it is a generalisation of the Klein-Gordon equation

$$\frac{\partial^2\phi(x,t)}{\partial t^2} - \frac{\partial^2\phi(x,t)}{\partial x^2} + \phi(x,t) = 0$$

which is a relativistic quantum wave equation for spin-0 particles. (But don't worry - we are not doing quantum field theory here). In this equation, $\phi(x,t)$ is a field variable; which we gave a physical interpretation as the angular displacement in our mechanical model. Fields possess energy and momenta, not unlike electromagnetic fields. However, the exact nature of these conserved quantities may differ. Note that you do not have to worry about what appears to be inconsistent units - the equation is **non-dimensionalised**.

Continue to make reference to the pendulum model whenever necessary.

(e) Work with the Klein-Gordon equation. Using the ansatz $\phi(x,t) = Ae^{i(kx\pm\omega t)}$, obtain the **dispersion relation** $\omega(k)$. [1]

These are still linear waves with a well-defined dispersion relation. The waves propagate independently and at different speeds; hence wavepackets spread out over time. To better understand nonlinear waves, let us study the soliton solutions of the Sine-Gordon equation.

(f) Show that $\phi_s(x) = 4\arctan e^{\pm x}$ is a solution to the Sine-Gordon equation for a **static** soliton in equilibrium. To simplify things, proving for just the plus case $\phi_s(x) =$ [2]

$4 \arctan e^x$ is sufficient. Sketch a graph of $\phi_s(x)$ for both cases **on the same graph** and label the asymptotes.
Hint: $\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$

From here on, we will use **kink** and **anti-kink** to describe the plus and minus soliton solutions in (f) respectively.

(g) Find the energy E_0 of a static kink. [3]
Hint: Energy is also non-dimensionalised; do not use constants from the mechanical model such as m and I in your answers.

(h) i. Let x' and t' be the coordinates in a frame moving at relativistic speed v in the positive x-direction. Express x' and t' in terms of x and t. [½]

ii. In terms of $\phi(x, t)$, the field expressed in the moving frame $\phi'(x', t') = \phi(x, t)$, and the respective coordinates, state an equation which shows that the Sine-Gordan equation is invariant under a Lorentz transformation. [½]

From here on, you may assume the Sine-Gordan equation to be Lorentz invariant.

(i) Using your results in (f) and (h), find the solution $\phi_k(x, t)$ to a solitary kink propagating at speed v in the positive x-direction. [1]

(j) What is the energy E_v of a kink propagating at speed v? Express your answer in terms of E_0 and v. [1]

(k) Now, consider two static kinks separated by a very large distance D. By considering the energy density or otherwise, deduce and justify a scaling relation between the **interaction energy** ΔE and the distance of separation D. [3]

(l) Using your results in (k), deduce whether the interaction force of each pair below are attractive or repulsive. [1]

a. kink - kink
b. kink - antikink
c. antikink - antikink

Fundamental Physical Constants — Frequently used constants

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
speed of light in vacuum	c	299 792 458	m s^{-1}	exact
Newtonian constant of gravitation	G	$6.674\,30(15) \times 10^{-11}$	m^3 kg^{-1} s^{-2}	2.2×10^{-5}
Planck constant*	h	$6.626\,070\,15 \times 10^{-34}$	J Hz^{-1}	exact
	$\hbar$	$1.054\,571\,817\ldots \times 10^{-34}$	J s	exact
elementary charge	e	$1.602\,176\,634 \times 10^{-19}$	C	exact
vacuum magnetic permeability $4\pi\alpha\hbar/e^2c$	μ_0	$1.256\,637\,061\,27(20) \times 10^{-6}$	N A^{-2}	1.6×10^{-10}
vacuum electric permittivity $1/\mu_0 c^2$	ϵ_0	$8.854\,187\,8188(14) \times 10^{-12}$	F m^{-1}	1.6×10^{-10}
Josephson constant $2e/h$	K_J	$483\,597.848\,4\ldots \times 10^{9}$	Hz V^{-1}	exact
von Klitzing constant $\mu_0 c/2\alpha = 2\pi\hbar/e^2$	R_K	$25\,812.807\,45\ldots$	Ω	exact
magnetic flux quantum $2\pi\hbar/(2e)$	Φ_0	$2.067\,833\,848\ldots \times 10^{-15}$	Wb	exact
conductance quantum $2e^2/2\pi\hbar$	G_0	$7.748\,091\,729\ldots \times 10^{-5}$	S	exact
electron mass	m_e	$9.109\,383\,7139(28) \times 10^{-31}$	kg	3.1×10^{-10}
proton mass	m_p	$1.672\,621\,925\,95(52) \times 10^{-27}$	kg	3.1×10^{-10}
proton-electron mass ratio	$m_\mathrm{p}/m_\mathrm{e}$	$1836.152\,673\,426(32)$		1.7×10^{-11}
fine-structure constant $e^2/4\pi\epsilon_0\hbar c$	α	$7.297\,352\,5643(11) \times 10^{-3}$		1.6×10^{-10}
inverse fine-structure constant	α^{-1}	$137.035\,999\,177(21)$		1.6×10^{-10}
Rydberg frequency $\alpha^2 m_\mathrm{e} c^2/2h$	cR_∞	$3.289\,841\,960\,2500(36) \times 10^{15}$	Hz	1.1×10^{-12}
Boltzmann constant	k	$1.380\,649 \times 10^{-23}$	J K^{-1}	exact
Avogadro constant	N_A	$6.022\,140\,76 \times 10^{23}$	mol^{-1}	exact
molar gas constant $N_\mathrm{A}k$	R	$8.314\,462\,618\ldots$	J mol^{-1} K^{-1}	exact
Faraday constant $N_\mathrm{A}e$	F	$96\,485.332\,12\ldots$	C mol^{-1}	exact
Stefan-Boltzmann constant $(\pi^2/60)k^4/\hbar^3c^2$	σ	$5.670\,374\,419\ldots \times 10^{-8}$	W m^{-2} K^{-4}	exact
Non-SI units accepted for use with the SI				
electron volt (e/C) J	eV	$1.602\,176\,634 \times 10^{-19}$	J	exact
(unified) atomic mass unit $\frac{1}{12}m(^{12}\mathrm{C})$	u	$1.660\,539\,068\,92(52) \times 10^{-27}$	kg	3.1×10^{-10}

* The energy of a photon with frequency ν expressed in unit Hz is $E = h\nu$ in J. Unitary time evolution of the state of this photon is given by $\exp(-iEt/\hbar)|\varphi\rangle$, where $|\varphi\rangle$ is the photon state at time $t = 0$ and time is expressed in unit s. The ratio $Et/\hbar$ is a phase.