2026 SPOT Marker's Report

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Three Mechanics Problems

1. (a) This question was unfortunately poorly done, although it is basically a SJPO standard problem. Many students did not know that the force exerted should be proportional to the relative speed **squared** $F \propto (v_0 - v)^2$ and not $(v_0 - v)$. The correct answer is $\frac{v_0}{3}$. But we've also seen a wide range of interesting answers from v_0, $\frac{v_0}{2}$, $\frac{v_0}{4}$, $\frac{v_0}{5}$.

   (b) This question was again very poorly done. Only 2 students solved it correctly. Almost all students only considered the kinematics and naively assumed that the component of velocity directed at P_1 will become 0, thus arriving at the wrong answer of $\omega_2 = \omega_1 \cos\theta$.

   Unfortunately, this is wrong because the new pivot P_2 will exert a non-trivial impulse. It is possible to solve it by considering the initial COM velocity as $\vec{v_1} = \vec{\omega_1} \times \vec{OP}_1$ and final COM velocity as $\vec{v_2} = \vec{\omega_2} \times \vec{OP}_2$. Then, the impulse is $\Delta\vec{p} = m(\vec{v_2} - \vec{v_1})$ and the change in angular momentum about the COM is $I\vec{\omega_1} + \Delta\vec{p} \times \vec{OP}_2 = I\vec{\omega_2}$. Substituting in the explicit expression for $\Delta\vec{p}$, we get

   $$I\vec{\omega_1} + m(\vec{\omega_2} \times \vec{OP}_2 - \vec{\omega_1} \times \vec{OP}_1) \times \vec{OP}_2 = I\vec{\omega_2}$$

   and we can subsequently solve for ω_2 in terms of ω_1.

   Of course, we did not expect you guys to do all these for a 3 mark question. The trick lies in realising that **angular momentum is conserved about** P_2, since contact forces do not exert any torque about P_2. The initial angular momentum can be decomposed into the angular momentum of the COM about P_2 and the angular momentum of the rigid body about its COM, while the final angular momentum is simple. I will leave it to you to work it out.

   (c) A handful of students managed to solve this part correctly (up to a sign error). This was taken (and slightly re-phrased) from a question from IZhO (International Zhautykov

Olympiad). Many considered conservation of angular momentum about the instantaneous point where the string is tangent to the cylinder. Unfortunately, this point is accelerating (and is also not the COM) so the naive angular momentum conservation does not apply.

Some students managed to identify the correct conserved quantity to be energy, since tension is always perpendicular to the velocity and does zero work. The rest will be left as an exercise to the reader.

Circuits

2. (a) Almost everyone got this part correct.

   (b) Unfortunately, only one person identified the intended idea. (interestingly this person was also the only one who got 2a wrong) The intended approach is to consider the "phasor" in the V-I graph of the nonlinear element - by using Kirchhoff's Law and parameterising the V-I relation using the phasor angle, you can derive that the phasor swings anticlockwise with **constant angular velocity** - hence the time is simply given by $\frac{\pi}{2\omega}$. This also makes latter parts very straightforward. Many students obtained U in terms of I and simply integrated. A handful of correct answers were obtained this way, but unfortunately many students failed to recognise that the final voltage of the capacitor is $2U_0$ - the nonlinear element allows the capacitor to charge up to a voltage beyond that of the battery, due to the negative V-I characteristic of the nonlinear element. Such devices are called active components, seen in some diodes or transistors.

   (c) Only 2 students obtained fully correct answers for this part. A handful of students obtained the correct expression $P = U_0 I_0 \sin\phi(1 - \cos\phi)$, but there were many mathematical errors when performing the differentiation. Some students were confused and simply took the potential difference across the capacitor to be the potential difference across the nonlinear element, obtaining relations like $\sin\phi\cos\phi$ instead. Finding the time t was too much for many students due to the mathematical intensity required to perform the integration again; but knowing the idea of constant angular velocity, this becomes much more straightforward.

   (d) The idea behind this question is to realise that equilibrium is achieved only when there is **no current** - this occurs at both ends of the nonlinear V-I graph. But the $(U_0, 0)$ end of the graph is the unstable equilibrium; with any small perturbation that results in current flow, the circuit is "activated" - the phasor swings anticlockwise with constant angular velocity and brings the system to the other point of equilibrium $(-U_0, 0)$. And since the phasor swings the entire π, the time is simply $\frac{\pi}{\omega}$.

Optics

3. This question is generally well done. Many students got the answer correct (up to a sign error in the exponent), and some put up a valiant attempt using Fermat's principle. Unfortunately, Fermat's principle does not directly apply here because it was not stated in the question that the light rays will necessarily focus at a single point.

Magnetic Lens

4. (a) This part was surprisingly done very poorly. The question gives you cylindrical symmetry; hence the correct approach is to construct a cylindrical gaussian surface of radius r and height Δz, taking the limit $\Delta z \to 0$ to obtain the required relation. Many students used the differential form of Gauss' Law $\nabla \cdot B = 0$ and expanded in cylindrical coordinates; you would obtain $\frac{1}{r}\frac{\partial(rB_r)}{\partial r} + \frac{\partial B_z}{\partial z} = 0$ assuming cylindrical symmetry, but

this is unfortunately insufficient. There is an additional condition implied in the question statement that r is small ("near the central axis") - this allows us to claim that the flux through the flat side of the cylindrical surface is $\pi r^2 B_z(z)$, assuming that B_z is approximately constant over the entire cylinder cross-section. This additional condition needs to be imposed to obtain meaningful results from the differential form, but no one who attempted this method identified the issue and simply tried to "scam" their way to a solution - one common mistake was stating $\frac{1}{r}\frac{\partial(rB_r)}{\partial r} + \frac{\partial B_z}{\partial z} = \frac{2}{r}B_r + \frac{\partial B_z}{\partial z} = 0$, but this is clearly wrong.

(b) Many students obtained $mr\ddot{\theta} = -ev_z B_r$ and went on to obtain the desired expression.

(c) Many students did not account for centrifugal force $mr\dot{\theta}^2$ - this will lead to answers off by a factor of 2. Most were able to convert $\ddot{r} = v^2\frac{d^2r}{dz^2}$. However, everyone made math errors subsequently and failed to obtain the correct value of k; a significant number of students even obtained dimensionally inaccurate values.

(d) Only a few students identified the correct idea. For the electrons to converge at the same point, $y(\phi)$ must be independent of the initial direction of the electrons - this means that we need to eliminate C_1 and C_2 from the given expression. And this is only possible for specific values of ϕ.

Weird Heat Engine

5. (a) This part is done poorly. Many students did not understand how the heat engine works and mistakenly thought that the point O' is rotating about O. Those who read the question properly managed to get the mark.

(b) All students were awarded some marks for this part. Everyone was able to see the substitution $V \to L$ and $P \to -\tau$. However, many missed out the minus sign and were awarded partial credits.

(c) This part is done well by many students who expanded the differential correctly. Many students attempted to reverse engineer the final expressions and were not given credits for doing that.

(d) Everyone who attempted this question was able to identify the isochoric and isothermal processes correctly as alluded to by the hint. However, many made the basic mistake of drawing points A and B to the left of C and D on the $\tau - L$ plot, although the rubber band at A, B is longer than that at C, D. Many also drew A, B at a lower vertical position than C, D, meaning that A, B have a lower τ value. However, a simple intuitive check should tell you that a more stretched rubber band should have greater tention.

Many students also did not carefully consider the exact shape of the isothermal lines. We can make use of the linear approximation $\rho = (\partial\tau/\partial L)_T$, which gives us $\tau = \rho L + C$ at constant temperature. Graphically, this means that both DA and BC should be connected with straight lines with the same gradient.

(e) A few students managed to solve this part correctly. There's nothing special. You just have to consider the heat input and output for each of the processes carefully. Many students managed to calculate the workdone correctly as the area enclosed, but got the heat input wrong because of sign errors.

Interference

6. (a) Many students missed out a factor of 2 in the path difference, as the light passes through the gas chamber twice in a round trip.

(b) The intended solution is to consider the most general case where the chamber is moving at high speeds and relativistic effects are not negligible. If we naively add the speed of light in the chamber $\frac{c}{n}$ to the speed of the chamber v to obtain the speed of light in the lab frame, we see that when $v > c\left(\frac{n-1}{n}\right)$, the light will be travelling faster than c! Because of how small $n-1$ is (as calculated in part (a)), the upper bound for v where the naive velocity addition stops working is actually quite small (by relativistic standards) at about 10^5 m s^{-1}.

There are many ways to go about doing this relativistically. You can work out the time and distance taken for the light to pass through the chamber in the chamber frame ($t' = \frac{nD}{c}$, $x' = D$) and Lorentz transform into the lab frame. Or you can work directly in the lab frame by considering relativistic velocity addition and length contraction to account for the reduced length of the moving chamber.

There were also numerous solutions where one consider the light being doppler shifted in the frame of the moving chamber. The correct answer is

$$\Delta\phi = \frac{4\pi(n-1)}{\sqrt{1-\frac{v^2}{c^2}}}\frac{D}{\lambda}$$

One student managed to obtain the exact expression and was given full credit, while most students obtained an expression correct to zeroth or first order of $\frac{v}{c}$ and were given partial credits. Some students lost precious marks because they wrongly calculated the speed of light in a medium with refractive index n to be nc instead of $\frac{c}{n}$.

(c) A handful of students managed to solve this part correctly. This is a classic problem. Since $p = \hbar k$, as the neutron travels against gravity, p and k decrease. To relate the phase difference to this change in k, we know that $d\phi = k\ dx$. We then integrate from 0 to the maximum height reached by the neutron beam and back to zero.

(d) Students who attempted this correctly identified the graph as an interference pattern with constructive interference at $\Delta\phi = 2\pi N$ with N being an integer, and destructive interference at $\Delta\phi = (2N+1)\pi$. However, no one obtained full credit as the intensity I is also proportional to the total energy per unit time: $I \propto \alpha$. So the equation of the graph would look like something like

$$I \propto \alpha \cos^2\left(2\alpha^{1/2} - \frac{4}{3}\alpha^{3/2}\right)$$

It will be an interference pattern with linearly increasing amplitude and tighter fringes.

Kinetic particle theory, planck's law and boltzmann distribution

7. (a) This part was very poorly done. Many came up with very creative reasons as to how the factor of $\frac{1}{4}$ comes about, and it was an amusing read for the markers. Some of the interesting (but incorrect) answers are included below.

    1. A photon has 4 degrees of freedom: one energy and three components of momentum, so it should be $\frac{1}{4}$!
    2. There's a factor of $\frac{3}{2}$ in $\frac{3}{2}k_BT$. And a cube has 6 faces, so $\frac{3}{2} \div 6 = \frac{1}{4}$, so it is $\frac{1}{4}$!
    3. A photon has energy $3k_BT$ with 6 degrees of freedom (not completely wrong, the average photon energy for a Planck's spectrum is about 2.7 k_BT). Then, $\gamma = 1+\frac{2}{f}$, and therefore $\gamma = \frac{4}{3}$ if $f = 6$. And $\frac{\gamma-1}{\gamma} = \frac{1}{4}$. Is this black magic?
    4. The projected area of a sphere is πr^2, and the total area of the sphere is $4\pi r^2$. The ratio gives $\frac{1}{4}$!
    5. Sneakily replacing random numbers with 4 and scamming their way through.

A handful of students put up a decent attempt at actually working out the photon flux through the cavity wall using integration. Consider a small area $\mathrm{d}A$ on the cavity wall. In time $\mathrm{d}t$, the volume of particles hitting the wall at an angle θ with respect to the normal is $\mathrm{d}A\, c\cos\theta\, \mathrm{d}t$. Note that we can use this because all photons, regardless of their frequencies (or energy), travel at the same speed. Assuming isotropy, the proportion of particles moving at angle θ is $n\sin\theta\,\mathrm{d}\theta/2$ where n is the number of particles per unit volume. Hence, the number of particles hitting the area per unit time per unit area is

$$\frac{\mathrm{d}N}{\mathrm{d}A\,\mathrm{d}t} = \frac{1}{2}nc\cos\theta\sin\theta\,\mathrm{d}\theta$$

Integrating this across θ from 0 to $\pi/2$ gives us $nc/4$. This proves the desired relation upon multiplying both sides by the average energy per particle.

(b) This part was well done. Most students matched the 3 rates correctly.

(c) Many students were able to express the rates $\dot{N}_a$ or $\dot{N}_b$ correctly and equate them to 0 for equilibrium. However, only a handful of students realise the second important condition that the population N_a and N_b must satisfy the Boltzmann distribution

$$\frac{N_b}{N_a} = e^{-\beta\hbar\Delta\omega}$$

Special relativity

8. (a) The key physical insight to minimise neutrino energy is that the velocities of the products in the center of momentum frame should be 0. Any additional velocity in the center of momentum frame can be seen as "wasted" energy. This was obtained by a handful of students. Many of these students went on to use 4-vectors; only a few obtained the correct answer, many made calculation errors.

There is a very elegant approach - consider the relativistic invariant before the reaction in the lab frame, and after the reaction *but in the center of momentum frame.* The relativistic invariant is both conserved and invariant across frames; and in the center of momentum frame, the invariant is simply the rest mass energy of both products due to the condition that the products have zero velocity in the center of momentum frame! This approach significantly reduces the compu

Quite a number of students were able to correctly write out equations for conservation of energy and momentum; these students were awarded partial credit.

(b) No one got any marks for this part.

Cavitation

9. (a) Many students correctly calculated v_{gas} with the ideal gas law. But only a handful were able to identify the correct approximation to neglect v_{liquid} - since liquids are much denser than gases. This was key to obtaining a solvable differential equation to obtain the correct answer.

(b) There are two contributions - surface energy $4\pi R^2\sigma$, which can be seen as "internal energy", and the pressure-volume energy due to the driving pressure, $V\Delta P$. Since the process is reversible, there is no entropy change. No one was able to correctly state all three terms, though some obtained partials.

(c) This is straightforward if part b was done correctly - simply differentiate $\Delta G_b(R)$ and equate to 0.

(d) Most students who attempted this part were able to obtain the correct answer. A few students went astray trying to expand out $\frac{\partial}{\partial r}(r^2 v_r)$ given in the question - the physical insight is that $r^2 v_r$ is constant if its spatial derivative equals 0 everywhere, there is no need to perform complex mathematics here. Or you can simply see it as $Av =$ constant.

(e) Unfortunately, no one was able to fully answer this part. A few students made valiant attempts, identifying the correct approach of energy conservation. Even fewer students noticed that you had to integrate the kinetic energy over all the liquid outside the bubble, and those who did were unable to perform the required calculations correctly. There are 3 main contributions to the total energy; kinetic energy of the liquid outside the bubble, surface energy and the driving work by ΔP - which were accounted for in the Gibbs Free Energy earlier in part b. Finding the correct total energy and differentiating by time would lead you to the required expression.

(f) Only one student got this part correct. The key idea is that at equilibrium, $\Delta P = P_{sat} - P_{ext}$ is balanced by Young-Laplace pressure $\frac{2\sigma}{R}$ - and P_{sat} can be found using the Clausius-Clapeyron relation obtained in part a.

Solitons

10. (a) A handful of students obtained the correct answer. There were many answers, however, which were dimensionally inconsistent. Many students forgot to multiply by a factor of b^2. The b^2 arises when making the continuous assumption; $\tau = K(\phi_{i+1} - \phi_i) + K(\phi_{i-1} - \phi_i) = Kb\left[\frac{\phi(x+b)-\phi(x)}{b} + \frac{\phi(x-b)-\phi(x)}{b}\right] = Kb^2 \frac{\frac{d\phi(x)}{dx} - \frac{d\phi(x-b)}{dx}}{b} = Kb^2 \frac{d^2\phi}{dx^2}$. Most approaches were not very mathematically rigorous, which was likely what led to such pitfalls. It is not difficult to spend a few seconds or so checking dimensions... such mistakes can be easily avoided.

(b) Most students who attempted this part obtained the correct answer.

(c) Most students who attempted this part obtained the correct answer. However, there were a few who wrote gravitational torque as $d \sin\phi$ instead of $mgd \sin\phi$, or gave the wrong sign for the torque - it should be negative since it is a restoring torque.

(d) A handful of students correctly identified that the restoring torque from gravity simply reduces the propagation speed; the maximum speed is just the result derived in part b. Physically, the propagation speed is limited by the *coupling strength* between successive spokes, given by the torsion K.

(e) Most students who attempted this part obtained the correct answer.

(f) Only 3 students obtained full credit for this part. Quite a few students could compute $\frac{\partial^2\phi(x)}{\partial x^2}$ correctly, but many struggled with $\sin\phi(x) = \sin(4\arctan e^x)$; the key is to expand the expression using double angle formulas. Most students who attempted to draw the graph drew it correctly.

(g) No one was able to make progress for this part. The key observation to make is that the mechanical model tells us what the energy should be - $\frac{1}{2}\frac{\partial\phi}{\partial t}^2$ for kinetic energy, $\frac{1}{2}\frac{\partial\phi}{\partial x}^2$ for torsional energy, and $1 - \cos\phi$ for gravitational potential energy. This can be expressed as an energy density which is integrated over the entire kink to find the total energy $E_0 = 8$. In field theory, the Hamiltonian is used to compute the energy of the field ϕ; but that is clearly out of syllabus and actually field theory, hence the analogous mechanical model was provided for physical insight.

(h) Most students could quote the Lorentz Transformation correctly. The original question was to derive the Lorentz invariance of the Sine-Gordon equation, but we realised that it would be too mathematically involved; hence it was greatly simplified.

(i) At this point, many students probably ran out of time; only one student got this correct. The idea here is that since the equation is Lorentz invariant, propagating kinks also obey the laws of special relativity - a kink moving with speed v is simply a Lorentz-boosted static kink! We can hence use the given solution for a static kink in part f and transform the argument to get the solution for a moving kink.

(j) Again using the relativistic analogy, it is simply γE_0. You can actually rigorously prove using the energy density with the approach from part g, as well as the solution for a moving kink, that γE_0 is the correct expression for energy. But the intention was to further the relativistic analogy.

(k) This part is very difficult. The key is to consider the energy density **in between** the two kinks - this is the point where the energy density is the most affected by changes in position of the kinks. You will be able to obtain that the interaction energy is proportional to e^{-D}.

(l) One student guessed the correct answer for this question. It is simply repulsive, attractive, repulsive - the kinks and antikinks act as **topological charges** which follow the same interaction laws as electrical charges. Fascinating stuff.