SPhO 2025 Questions (UNOFFICIAL)

FIZIKA

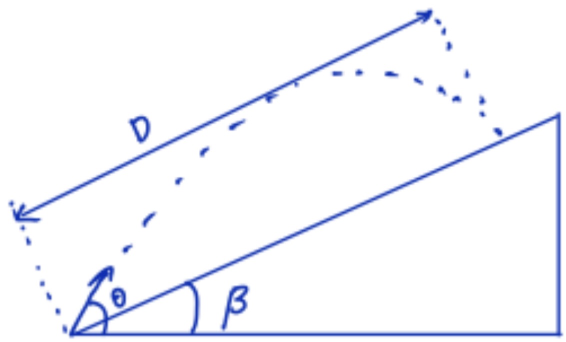
October 2025

We thank FIZIKA students who have sat for SPhO 2025 for compiling the questions for us.

1 Question 1 [7]

1.1 Question

A projectile is shot up a slope of incline angle β to the x-axis at an initial velocity v_0 and angle θ to the x-axis, where $\theta > \beta$. The slope and the projectile both start at the origin. Express all answers in terms of v_0, β, θ and g.



(i) Find the time taken for the projectile to land back on the slope. [3]

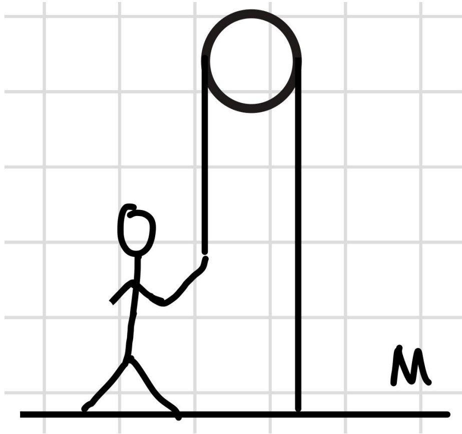
(ii) Find the distance up the slope it has travelled. [2]

(iii) Prove that for the maximum distance, $-\cot 2\theta = \tan\beta$. [2]

2 Question 2 [14]

2.1 Question

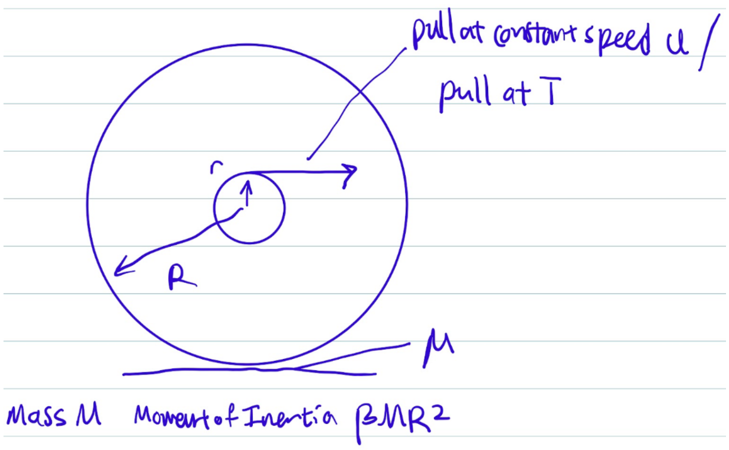
A man is on an elevator held up by a rope on a pulley. The other end of the rope is getting tugged on by the man in the elevator. The man and the elevator have a total mass of M. Refer to the figure below.



(i) Find the minimum force the man needs to apply to move upward. [2]

(ii) Say the elevator moves a height h upwards. By considering the change in GPE, comment on the conservation of energy. [2]

I have a yo-yo (see figure) of moment of inertia $I = bMR^2$ about its central axis. The coefficient of friction is μ.



(iii) If I pull the string at constant speed u, find the CM velocity, assuming no slipping. [2]

(iv) Now, I pull the string at constant tension T. Show that the condition for friction to point left is $r < bR$, still assuming no slipping. [3]

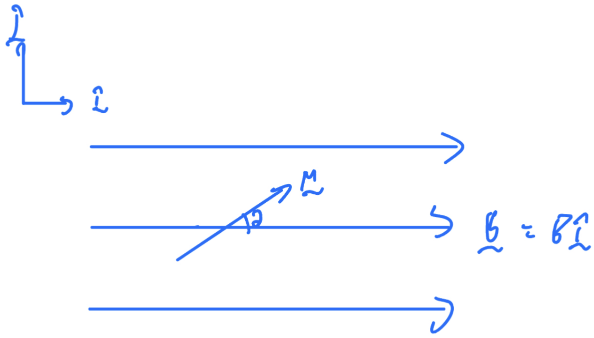
(v) Consider that there is slipping now. What is the minimum T required for friction to point left? [3]

(vi) Consider the limit $r \to bR$, and explain why your answer makes sense. [2]

3 Question 3 [10]

3.1 Question

A magnetic dipole moment of μ makes an angle θ with the x-axis, and is placed in a uniform, constant magnetic field $\mathbf{B} = B\hat{\mathbf{i}}$. Suppose that it has a moment of inertia I about the z-axis.



(i) State the axis of rotation of the dipole. [1]

(ii) Find the torque on the magnetic dipole, as a vector in Cartesian coordinates. [2]

(iii) Find the work done by the torque as the dipole rotates from $\theta_\mathrm{i} = \frac{\pi}{4}\,\mathrm{rad}$ to $\theta_\mathrm{f} = 0\,\mathrm{rad}$, and find the change in the potential energy. [2]

(iv) Show that for small θ, the motion of the dipole is simple harmonic, and obtain an expression for the period of oscillations. [3]

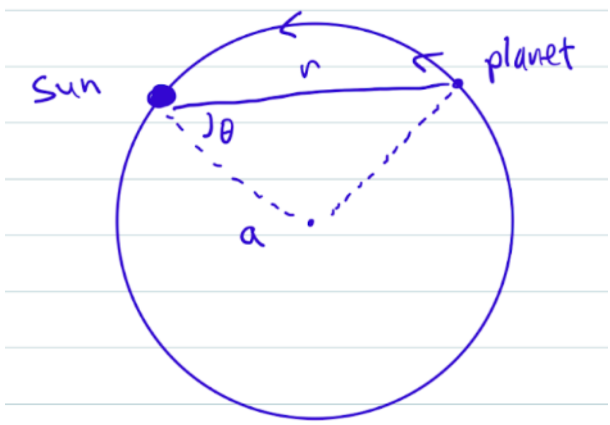
(iv) Suppose $\mathbf{B}$ now varies with y-position (increasing with increasing $\hat{\mathbf{j}}$ component). State whether there will be a net force and a net torque on the dipole. [2]

4 Question 4 [10]

4.1 Question

(i) Given a central force is such that $F(r) = m\left(\ddot{r} - r\dot{\theta}^2\right)$, by using the substitution $u = \frac{1}{r}$, prove the Binet equation $F(u) = -\frac{L^2u^2}{m}\left(\frac{\mathrm{d}^2u}{\mathrm{d}\theta^2} + u\right)$, where L is the angular momentum. [5]

(ii) A planet experiences a central force from the Sun and moves around in a circular orbit of radius a, as per the figure below:

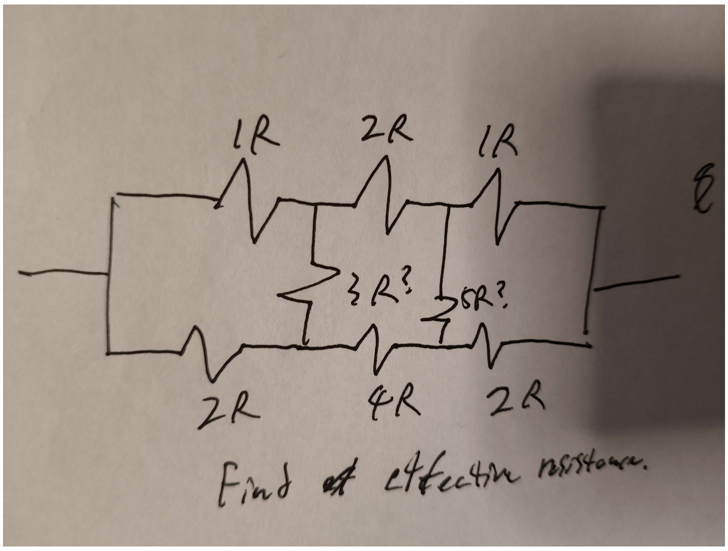


If $F(r) = -Cr^n$, where C is a constant, find the value of n. [5]

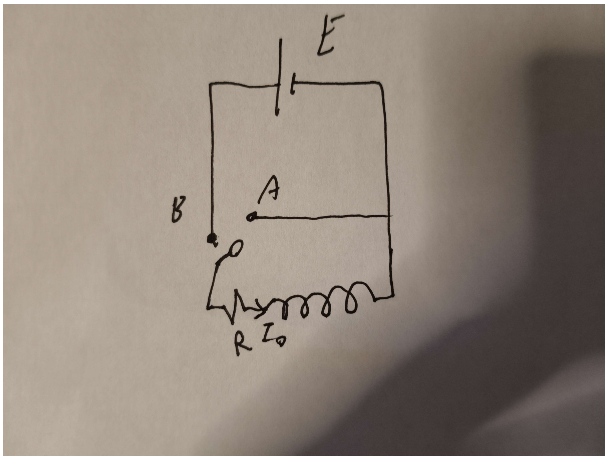
5 Question 5 [10]

5.1 Question

(i) Find the effective resistance across the terminals of the circuit below. [2]



(ii) Consider the RL circuit below with resistance R, inductance L and a battery of emf E. The switch is initially at position A. I_0 is the initial current through the circuit, and $I_0 < \frac{E}{R}$. At time $t = 0$, the switch is flipped to position B. Show that the new current in the circuit at time Δt after flipping the switch is given by $\left(I_0 - \frac{E}{R}\right) \exp\left(-\frac{R}{L}\Delta t\right) + \frac{E}{R}$. [3]



(iii) The switch is then flipped back to position A. Find the new current in the circuit at another time Δt after flipping the switch back. [2]

(iv) The switch is flipped back and forth A and B at a regular time intervals of Δt. Find the steady-state current at $t \to \infty$. [3]

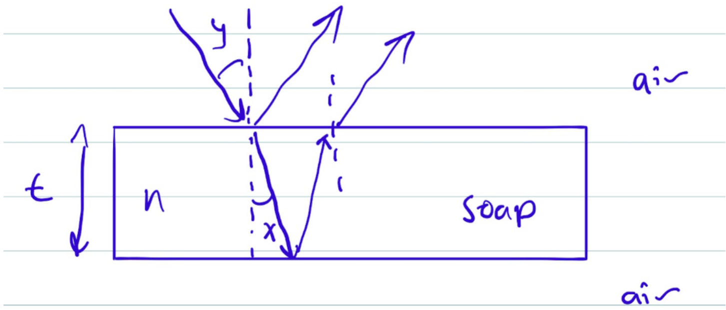
6 Question 6 [10]

6.1 Question

(i) Consider a cube of refractive index $n = 1.33$. Light enters from the top surface of the cube at its centre, at an angle of 45°. Determine whether light will exit through the side faces, and if so, determine the angle that it exits at. [2]

(ii) Consider a flat plate with n increasing linearly with thickness, from $n = 1$ at the top to $n = 2$ at the bottom. The plate is surrounded by a vacuum. If light enters at an angle of 45°, sketch the path of light in the medium. What angle does it exit at? [2]

(iii) Prove that the condition for constructive interference of this system in the figure below is $\frac{2t}{\cos x}(n - \sin x \sin y) = \left(m + \frac{1}{2}\right)\lambda$. [4]

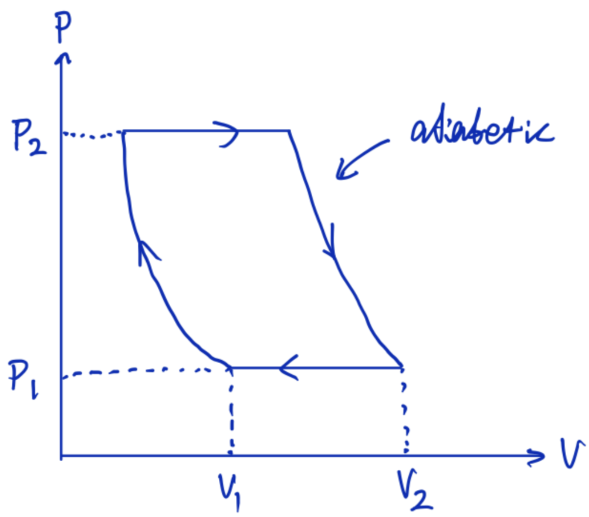


(iv) Using the numerical values $n = 1.33, t = 875\,\text{nm}$ and $y = 45°$, which wavelengths in the visible light spectrum will have constructive interference? [2]

7 Question 7 [10]

7.1 Question

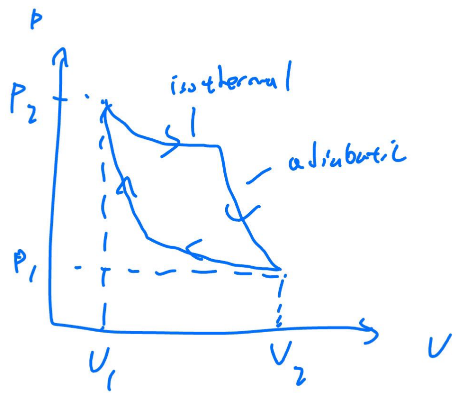
Consider the following $P - V$ diagram of the Brayton cycle, comprised of two isobaric processes and two adiabatic processes:



It is given that $P_2 = 3P_1$, $V_2 = 2V_1$, and the gas is monoatomic (hence $\gamma = \frac{5}{3}$).

(i) Show that the thermodynamic efficiency of the Brayton cycle is $\eta = 1 - \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}-1}$, and calculate its numerical value in this scenario. [5]

(ii) A Carnot cycle as shown in the figure below runs with the same pressure and volume values.



Find the thermodynamic efficiency of this Carnot cycle. [3]

(iii) You should have found that the efficiency of the Brayton cycle was higher than that of the Carnot cycle. Explain why the Brayton cycle can be more efficient than the Carnot cycle in this case. [2]

8 Question 8 [8]

8.1 Question

(i) Two radioactive species A and B are of the same initial amount. After 10 minutes, $N_A = 100N_B$. Given that twice the mean lifetime of A is equal to thrice the mean lifetime of B, find the mean lifetime of B. [4]

(ii) Prove that the de Broglie wavelength of a particle of rest mass m and kinetic energy K can be written as $\lambda = \frac{hc}{\sqrt{K(K+qmc^2)}}$, where q is a constant to be determined. [3]

(iii) The Compton wavelength is defined as $\lambda_c = \frac{h}{mc}$. For a particle with its kinetic energy being 3 times of its rest energy, find the value of $\frac{\lambda}{\lambda_c}$. [2]

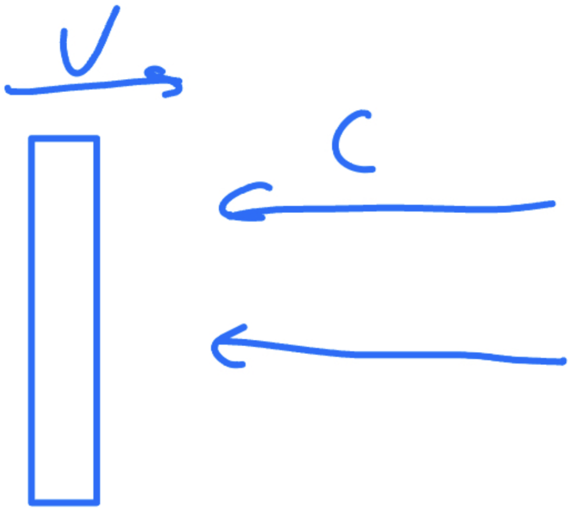
9 Question 9 [11]

9.1 Question

(i) State the Lorentz transformation of the 4-vector $(\omega, c\mathbf{k})$, where $|\mathbf{k}| = \frac{\omega}{c}$ is the wavevector/vector wavenumber of the wave. [2]

(ii) Prove that the quantity $\mathbf{k} \cdot \mathbf{r} - \omega t$ is Lorentz invariant, where $\mathbf{r}$ is position. [2]

A thin mirror moves at relativistic speed v to the right, and is bombarded by light rays going to the left. All light rays are incident normally on the mirror.



(iii) The frequency of the incoming photons is ω_{i} when measured in the lab frame. Find the angular frequency of the reflected photons ω_{r} in the lab frame. You may use $\beta = \frac{v}{c}$ to simplify your answer. [4] *Yes, ω here is defined as frequency, not angular frequency. Strange notation.*

(iv) Find the energy of one reflected photon. [1]

(v) The average incoming energy flux of photons is P_{i}. Find the average reflected energy flux P_{r}. [2]