USA TST Selection Test for 68^{th} IMO and 16^{th} EGMO

Greensboro, NC

Day I 12:30pm – 5:00pm

Saturday, June 20, 2026

Time limit: 4.5 hours. If you need to add page headers after the time limit, you must do so under proctor supervision. Proctors may not answer clarification questions.

You may keep the problems, but you cannot discuss them publicly until they are posted by staff online.

Problem 1. Let ABC be an acute scalene triangle. Let O be the circumcenter of ABC, D be the foot of the altitude from A to BC, and P be a point on line AO not lying on the circumcircle of ABC. The circle with diameter AP intersects the circumcircle of BCP at $E \neq P$, and the line through A parallel to BC at $F \neq A$. Prove that D, E, and F are collinear.

Problem 2. Given a graph G and a positive integer ℓ, an ℓ-*coloring* of G is a function $\sigma\colon V(G) \to \{1, 2, 3, \ldots, \ell\}$ such that for any edge uv in G, $\sigma(u) \neq \sigma(v)$. Two ℓ-colorings σ and σ' are *close* if $\sigma(u) \neq \sigma'(u)$ for at most one $u \in V(G)$.

Find all pairs of positive integers (k, m) with $k \leq m$ such that for all graphs G and pairs (τ, τ') of k-colorings of G, there exists a sequence $\pi_0, \pi_1, \ldots, \pi_n$ of m-colorings of G such that $\pi_0 = \tau$, $\pi_n = \tau'$, and π_i and π_{i+1} are close for all $0 \leq i < n$.

Note: $V(G)$ denotes the vertices of G. Since $k \leq m$, τ and τ' are also m-colorings of G.

Problem 3. Suppose $f_1, f_2, \ldots, f_{1000} \in \mathbb{C}[x]$ are single-variable polynomials with complex coefficients such that there do not exist distinct $i, j \in \{1, 2, \ldots, 1000\}$ and $a \in \mathbb{C}$ with $f_i = af_j$. Given that the degree of f_1 is 10, find the minimum possible total degree of

$$\sum_{k=1}^{1000} f_k(x_1)f_k(x_2)\cdots f_k(x_{2026}).$$

Note: For example, the total degree of the polynomial $g(x, y) = 20x^3y^4 + 26x^6$ is 7 because $7 = 3 + 4 > 6$.

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Day II 12:00pm – 4:30pm

Monday, June 22, 2026

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Problem 4. A *beehive* is a polygon with all internal angles being multiples of 60°. Find the least positive integer k for which every 2026-sided beehive can be expressed as the union of at most k convex beehives.

Note: A polygon contains its interior and boundary. Its boundary must be a single loop that does not intersect itself. Internal angles of 180° are not allowed.

Problem 5. Let $\Omega(k)$ denote the number of prime factors of k, counted with multiplicity. For example, $\Omega(20) = 3$, because $20 = 2 \cdot 2 \cdot 5$. Prove that for all positive integers n, there exists a positive integer k strictly between n^2 and $(n+1)^2$ such that $\Omega(k)$ is odd.

Problem 6. Let ABC be a triangle with circumcircle Γ and let S be a point inside triangle ABC. Let $P \neq S$ be the second intersection of line AS with the circumcircle of BCS.

Let $T \neq A$ be a variable point on Γ, and let $Q \neq A$ and $R \neq A$ be the second intersections of line AT with the circumcircles of ABP and ACP, respectively. Let ω be the circumcircle of PQR. Show that, as T varies along Γ, the intersection(s) of line ST with ω lie on two fixed circles (excluding cases where ω is undefined).

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Day III 12:00pm – 4:30pm

Wednesday, June 24, 2026

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Problem 7. Determine all ordered pairs of positive integers (a, b) such that $n+1$ divides $\binom{an+b}{n}$ for all positive integers n.

Problem 8. Let A be a finite set of positive real numbers. Define a_k to be the number of distinct positive real numbers that can be expressed as a sum of at most k not necessarily distinct elements of A. Prove that $2025a_{2026} \geq 2026a_{2025}$.

Problem 9. In the plane, we say that a set of points $A = \{A_1, A_2, \ldots, A_k\}$ *demolishes* a set of points $B = \{B_1, B_2, \ldots, B_\ell\}$ if, for all points P,

$$PA_1 + PA_2 + \cdots + PA_k \geq PB_1 + PB_2 + \cdots + PB_\ell.$$

Let S be a set of n points in the plane. Show that we can partition S into two sets A and B such that A demolishes B and $|A| \leq |B| + 100\sqrt{n}$.